



Robotics 2

Linear parametrization and identification of robot dynamics

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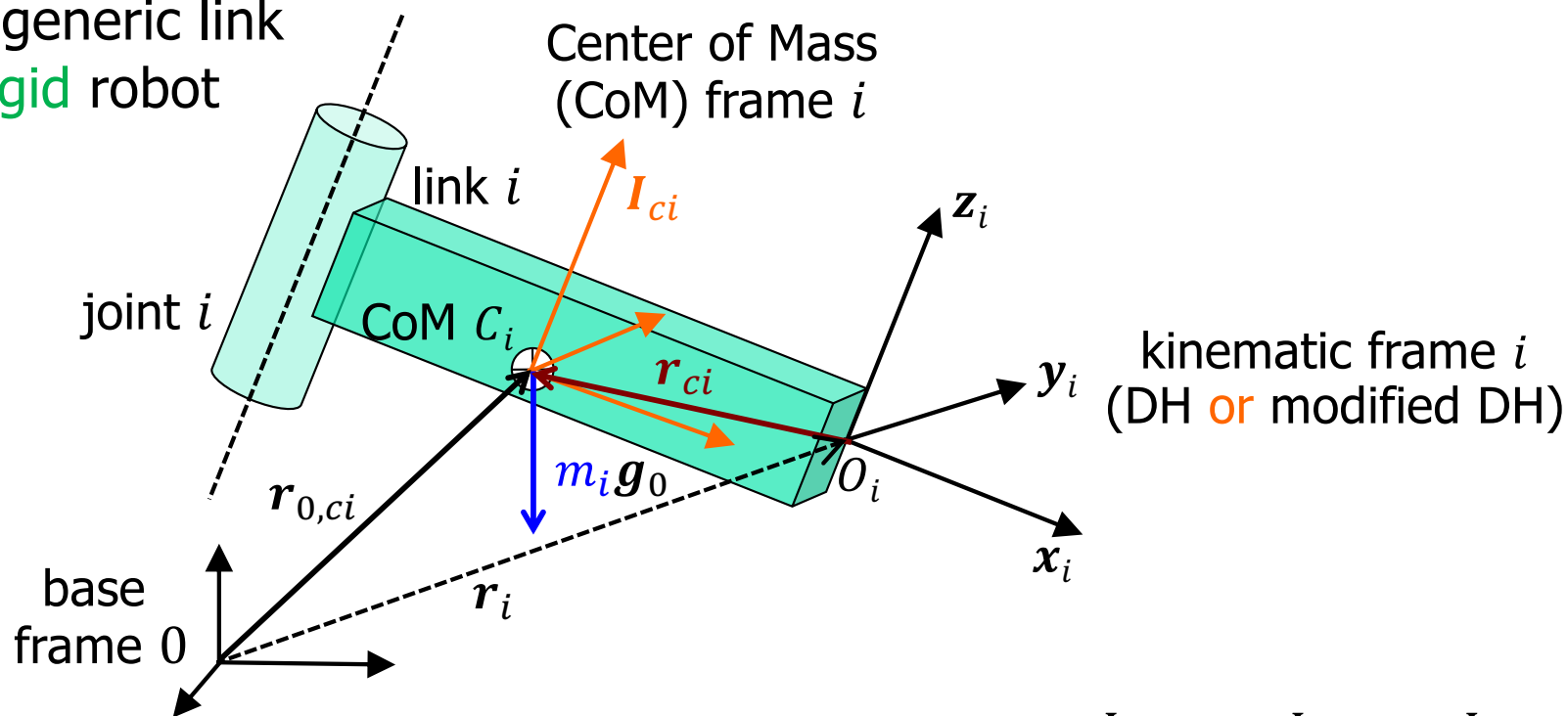


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Dynamic parameters of robot links

- consider a generic link of a **fully rigid** robot



- each link is characterized by 10 dynamic parameters
$$\left[\begin{array}{l} m_i \\ \mathbf{r}_{ci} = \begin{pmatrix} r_{xi} \\ r_{yi} \\ r_{zi} \end{pmatrix} \\ \mathbf{I}_{ci} = \begin{pmatrix} I_{ci,xx} & I_{ci,xy} & I_{ci,xz} \\ & I_{ci,yy} & I_{ci,yz} \\ \text{symm} & & I_{ci,zz} \end{pmatrix} \end{array} \right]$$
- however, robot dynamics depends **only on some** of these parameters and possibly in a **nonlinear** way (e.g., via the combination $I_{ci,zz} + m_i r_{xi}^2$)



Dynamic parameters of robots

- both the kinetic energy and the gravity potential energy can be rewritten so that a **new** set of dynamic parameters appears **only in a linear way**
 - need to re-express link inertia and CoM position in (any) **known** kinematic frame attached to the link (same orientation as the barycentric frame)

- fundamental kinematic relation

$$v_{ci} = v_i + \omega_i \times r_{ci} = v_i + S(\omega_i) r_{ci} = v_i - S(r_{ci}) \omega_i$$

- kinetic energy of link i

$$T_i = \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} \omega_i^T I_{ci} \omega_i \quad \Leftrightarrow \text{"reversing" König theorem now ...}$$

$$= \frac{1}{2} m_i (v_i - S(r_{ci}) \omega_i)^T (v_i - S(r_{ci}) \omega_i) + \frac{1}{2} \omega_i^T I_{ci} \omega_i$$

$$= \frac{1}{2} m_i v_i^T v_i + \frac{1}{2} \omega_i^T \underbrace{\left(I_{ci} + m_i S^T(r_{ci}) S(r_{ci}) \right)}_{\text{Steiner theorem}} \omega_i - v_i^T S(m_i r_{ci}) \omega_i$$

$$I_i = \begin{pmatrix} I_{i,xx} & I_{i,xy} & I_{i,xz} \\ & I_{i,yy} & I_{i,yz} \\ \text{symm} & & I_{i,zz} \end{pmatrix}$$



Standard dynamic parameters of robots

- gravitational potential energy of link i

$$U_i = -m_i g_0^T r_{0,ci} = -m_i g_0^T (r_i + r_{ci}) = -m_i g_0^T r_i - g_0^T (m_i r_{ci})$$

- by expressing vectors and matrices in frame i , both T_i and U_i will be **linear** in the set of 10 (constant) **standard parameters** $\pi_i \in \mathbb{R}^{10}$


$$T_i = \frac{1}{2} m_i {}^i v_i^T {}^i v_i + m_i {}^i r_{ci}^T S({}^i v_i) {}^i \omega_i + \frac{1}{2} {}^i \omega_i^T {}^i I_i {}^i \omega_i$$

$$U_i = -m_i g_0^T r_i - g_0^T {}^0 R_i (m_i {}^i r_{ci})$$

mass of link i
(0-th order moment)

mass \times CoM
position of link i
(1-st order moment)

inertia of link i
(2-nd order moment)

$$\pi_i = \begin{pmatrix} m_i \\ m_i {}^i r_{ci} \\ \text{vect}\{ {}^i I_i \} \end{pmatrix} = (m_i \quad m_i {}^i r_{ci,x} \quad m_i {}^i r_{ci,y} \quad m_i {}^i r_{ci,z} \quad {}^i I_{i,xx} \quad {}^i I_{i,xy} \quad {}^i I_{i,xz} \quad {}^i I_{i,yy} \quad {}^i I_{i,yz} \quad {}^i I_{i,zz})^T$$


- since the E-L equations involve only **linear** operations on T and U , also the robot dynamic model is linear in the standard parameters $\pi \in \mathbb{R}^{10N}$



Linearity in the dynamic parameters

- using a $N \times 10N$ regression matrix Y_π that depends only on **kinematic** quantities, the robot dynamic equations can always be rewritten **linearly** in the **standard dynamic parameters** as

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = Y_\pi(q, \dot{q}, \ddot{q}) \pi = u$$
$$\pi^T = (\pi_1^T \quad \pi_2^T \quad \cdots \quad \pi_N^T)$$

- the open kinematic chain structure of the manipulator implies that the i -th dynamic equation can depend only on the dynamic parameters of links from i to $N \Rightarrow Y_\pi$ has a **block upper triangular** structure

$$Y_\pi(q, \dot{q}, \ddot{q}) = \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ 0 & Y_{22} & \cdots & Y_{2N} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & Y_{NN} \end{pmatrix} \quad \begin{array}{l} \text{with row vectors} \\ Y_{ij} \text{ of size } 1 \times 10 \end{array}$$

Property: element m_{ij} of $M(q)$ depends at most on (q_{k+1}, \dots, q_N) , with $k = \min\{i, j\}$, and at most on the dynamic parameters of links h to N , with $h = \max\{i, j\}$



Linearity in the dynamic coefficients

- many standard parameters do not appear (“play no role”) in the dynamic model of a given robot \Rightarrow the associated **columns of Y_π are 0!**
- some standard parameters may appear only in fixed combinations with others \Rightarrow the associated **columns of Y_π are linearly dependent!**
- one can isolate $p \ll 10N$ **groups** of parameters π (associated to p **functionally independent columns Y_{indep}** of Y_π) and partition matrix Y_π in two blocks, the second containing dependent (or zero) columns as $Y_{dep} = Y_{indep}T$, for a suitable $p \times (10N - p)$ constant matrix T

$$\begin{aligned} Y_\pi(q, \dot{q}, \ddot{q}) \pi &= (Y_{indep} \quad Y_{dep}) \begin{pmatrix} \pi_{indep} \\ \pi_{dep} \end{pmatrix} = (Y_{indep} \quad Y_{indep}T) \begin{pmatrix} \pi_{indep} \\ \pi_{dep} \end{pmatrix} \\ &= Y_{indep}(\pi_{indep} + T\pi_{dep}) = \boxed{Y(q, \dot{q}, \ddot{q}) a} \end{aligned}$$

- these grouped parameters are called **dynamic coefficients $a \in \mathbb{R}^p$** , “the only that matter” in robot dynamics (= **base parameters** by W. Khalil)
- the **minimal number p** of dynamic coefficients that is needed can also be checked numerically (see \rightarrow identification)



Linear parametrization of robot dynamics

it is **always** possible to rewrite the dynamic model in the form

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = Y(q, \dot{q}, \ddot{q}) a = u$$

regression matrix a = vector of dynamic coefficients

$N \times p$

$p \times 1$

e.g., the **heuristic** grouping (found by inspection) on the planar 2R robot

$$\begin{pmatrix} \ddot{q}_1 & c_2(2\ddot{q}_1 + \ddot{q}_2) - s_2(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) & \ddot{q}_2 & c_1 & c_{12} \\ 0 & c_2\dot{q}_1 + s_2\dot{q}_1^2 & \dot{q}_1 + \dot{q}_2 & 0 & c_{12} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{aligned} a_1 &= I_{c1,zz} + m_1 d_1^2 + I_{c2,zz} + m_2 d_2^2 + m_2 l_1^2 \\ a_2 &= m_2 l_1 d_2 \\ a_3 &= I_{c2,zz} + m_2 d_2^2 \\ a_4 &= g_0(m_1 d_1 + m_2 l_1) \\ a_5 &= g_0 m_2 d_2 \end{aligned}$$

Note: 4 more coefficients are added when including the coefficients $F_{V,i}$ and $F_{C,i}$ of viscous and Coulomb friction at the joints (“decoupled” terms appearing only in i –th equation, for $i = 1,2$)



Linear parametrization of a 2R planar robot ($N = 2$)

- being the kinematics known (i.e., l_1 and g_0), the number of dynamic coefficients can be reduced since we can merge the two coefficients $a_2 = m_2 l_1 d_2$ & $a_5 = g_0 m_2 d_2 \Rightarrow a_2 = m_2 d_2$ (factoring out l_1 and g_0)
- therefore, after regrouping, **$p = 4$ dynamic coefficients** are sufficient

$$\begin{pmatrix} \ddot{q}_1 \\ 0 \end{pmatrix} \begin{pmatrix} l_1 c_2 (2\ddot{q}_1 + \ddot{q}_2) - l_1 s_2 (\dot{q}_2^2 + 2\dot{q}_1 \dot{q}_2) + g_0 c_{12} \\ l_1 (c_2 \ddot{q}_1 + s_2 \dot{q}_1^2) + g_0 c_{12} \end{pmatrix} \begin{pmatrix} \ddot{q}_2 \\ \ddot{q}_1 + \ddot{q}_2 \end{pmatrix} \begin{pmatrix} g_0 c_1 \\ 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = Y a = u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$a_1 = I_{c1,zz} + m_1 d_1^2 + I_{c2,zz} + m_2 d_2^2 + m_2 l_1^2$$

$$a_3 = I_{c2,zz} + m_2 d_2^2$$

$$a_2 = m_2 d_2$$

$$a_4 = m_1 d_1 + m_2 l_1$$

- this (minimal) linear parametrization of robot dynamics is **not unique**, both in terms of the chosen set of dynamic coefficients a and for the associated regression matrix Y
- a systematic procedure for its derivation would be preferable



Linear parametrization of a 2R planar robot ($N = 2$)

- as alternative to the previous heuristic method, apply the **general procedure**
 - $10N = 20$ **standard parameters** π are defined for the two links
 - from the assumptions made on CoM locations, **only 5** such parameters actually appear, namely (with ${}^i r_{ci,x} = -l_i + d_i$)

$$\text{link 1: } \underbrace{m_1 d_1}_{\pi_1} \quad \underbrace{I_{1,zz}}_{\pi_2} = I_{c1,zz} + m_1 d_1^2 \quad \text{link 2: } \underbrace{m_2}_{\pi_3} \quad \underbrace{m_2 d_2}_{\pi_4} \quad \underbrace{I_{2,zz}}_{\pi_5} = I_{c2,zz} + m_2 d_2^2$$

- in the 2×5 matrix Y_π , the 3rd column (associated to m_2) is $Y_{\pi_3} = Y_{\pi_1} l_1 + Y_{\pi_2} l_1^2$
- after regrouping/reordering, **$p = 4$ dynamic coefficients** are again sufficient

$$\begin{pmatrix} g_0 c_1 & \ddot{q}_1 & l_1 c_2 (2\ddot{q}_1 + \ddot{q}_2) - l_1 s_2 (\dot{q}_2^2 + 2\dot{q}_1 \dot{q}_2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 \\ 0 & 0 & l_1 (c_2 \ddot{q}_1 + s_2 \dot{q}_1^2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = Y a = u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$a_1 = m_1 d_1 + m_2 l_1 \quad a_2 = I_{1,zz} + m_2 l_1^2 = (I_{c1,zz} + m_1 d_1^2) + m_1 l_1^2 \quad a_3 = m_2 d_2 \quad a_4 = I_{2,zz} = I_{c2,zz} + m_2 d_2^2$$

- determining a **minimal parameterization** (i.e., minimizing p) is important for
 - experimental identification of dynamic coefficients
 - adaptive/robust control design in the presence of uncertain parameters



Identification of dynamic coefficients

- in order to “use” the model, one needs to know the numeric values of the robot **dynamic coefficients**
 - robot manufacturers provide at most only a few principal dynamic parameters (e.g., link masses)
- **estimates** can be found with CAD tools (e.g., assuming uniform mass)
- friction coefficients are (slowly) varying over time
 - lubrication of joints/transmissions
- for an added payload (attached to the E-E)
 - a change in the 10 dynamic parameters of last link ...
 - ... implies a variation of (almost) all robot dynamic coefficients!
- preliminary **identification experiments** are needed
 - robot **in motion** (dynamic issues, not just static or geometric ones!)
 - **only** the robot dynamic **coefficients** can be identified (and **not all** the link standard parameters!)



Identification experiments

1. **choose** a motion trajectory $q_d(t)$ that is sufficiently “exciting”, i.e.,
 - explores the robot workspace and involves all components in the robot dynamic model
 - is periodic, with multiple frequency components
2. **execute** this motion (approximately) by means of a control law
 - taking advantage of any available information on the robot model
 - often $u = K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$ (PD, **no model** information used)
3. **measure** q (encoders) in n_c time instants (and, if available, also \dot{q})
 - joint velocity \dot{q} and acceleration \ddot{q} can be estimated later **off line** by numerical differentiation (use of **non-causal** filters is feasible)
4. with such measures/estimates, **evaluate** the regression matrix Y (on the left) and use the applied commands u (on the right) in the expression

$$Y(q(t_k), \dot{q}(t_k), \ddot{q}(t_k)) a = u(t_k) \quad k = 1, \dots, n_c$$



Least Squares (LS) identification

- set up the system of **linear** equations

$$n_c \times N \begin{matrix} \uparrow \\ \downarrow \end{matrix} \begin{pmatrix} Y(q(t_1), \dot{q}(t_1), \ddot{q}(t_1)) \\ \vdots \\ Y(q(t_{n_c}), \dot{q}(t_{n_c}), \ddot{q}(t_{n_c})) \end{pmatrix} a = \begin{pmatrix} u(t_1) \\ \vdots \\ u(t_{n_c}) \end{pmatrix} \iff \bar{Y} a = \bar{u}$$

- sufficiently “exciting” trajectories, large enough number of samples ($n_c \times N \gg p$), and their suitable selection/position guarantee that **rank**(\bar{Y}) = p (full column rank)
- solution by **pseudoinversion** of matrix \bar{Y}

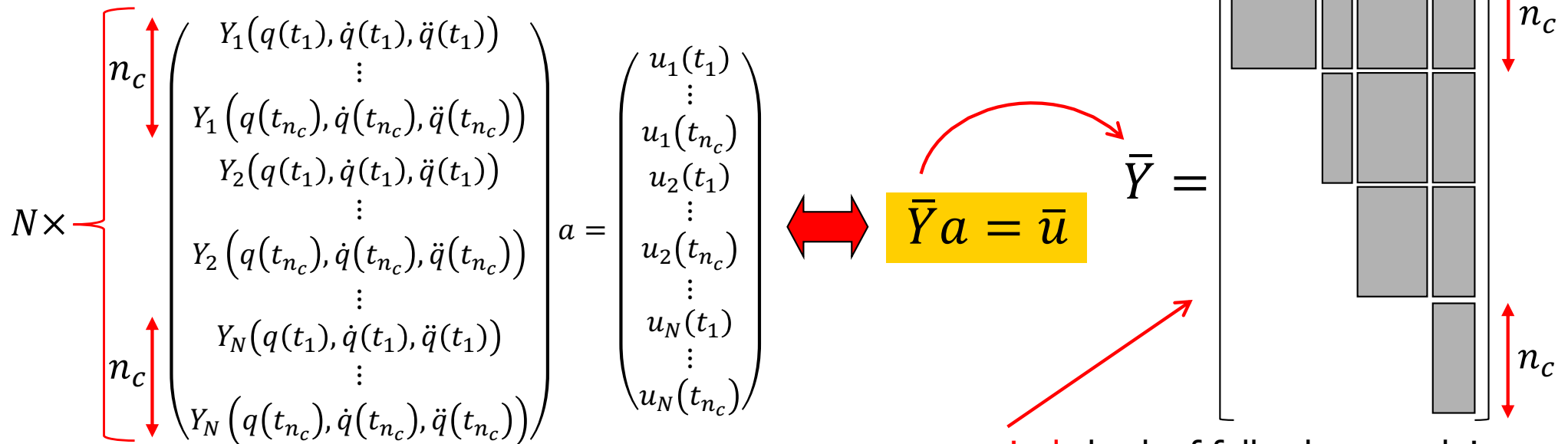
$$a = \bar{Y}^\# \bar{u} = (\bar{Y}^T \bar{Y})^{-1} \bar{Y}^T \bar{u} \quad (\in \mathbb{R}^p)$$

- one can also use a **weighted** pseudoinverse, to take into account different levels of noise in the collected measures



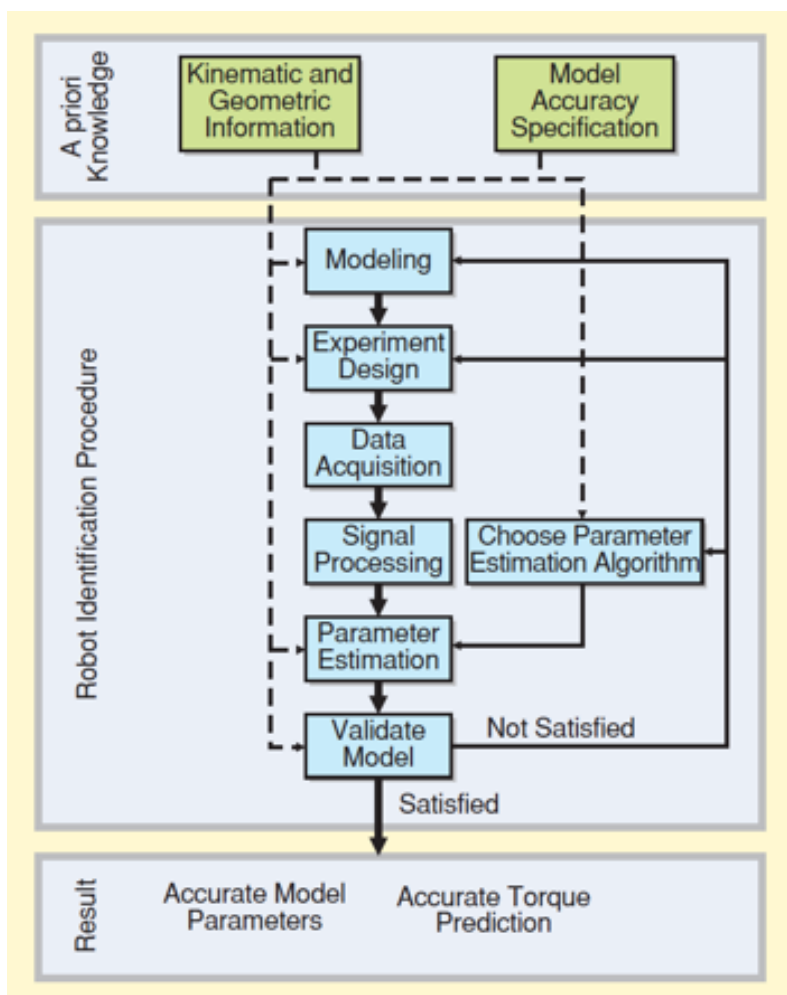
Additional remarks on LS identification

- it is convenient to preserve the **block (upper) triangular structure** of the regression matrix, by “stacking” all time evaluations **in row by row sequence** of the original Y matrix

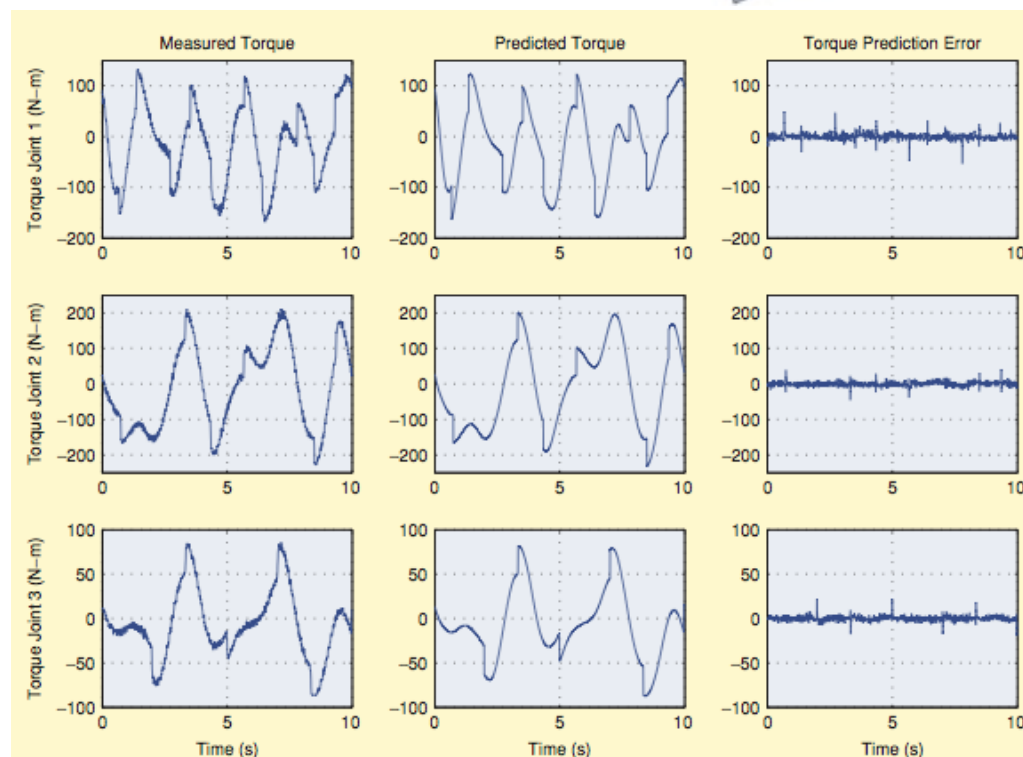


- further practical hints
 - numerical** check of full column rank is more robust \Leftrightarrow **rank** = p (# of col's)
 - outlier data** can be eliminated in advance (when building Y)
 - if sufficiently rich **friction** models are not included in Ya , **discard the data** collected at joint velocities **close to zero**

Summary on dynamic identification



KUKA IR 361 robot and optimal excitation trajectory



results after identification (first three joints only)

J. Swevers, W. Verdonck, and J. De Schutter:
 "Dynamic model identification for industrial robots"
 IEEE Control Systems Mag., Oct 2007

Dynamic identification of KUKA LWR4



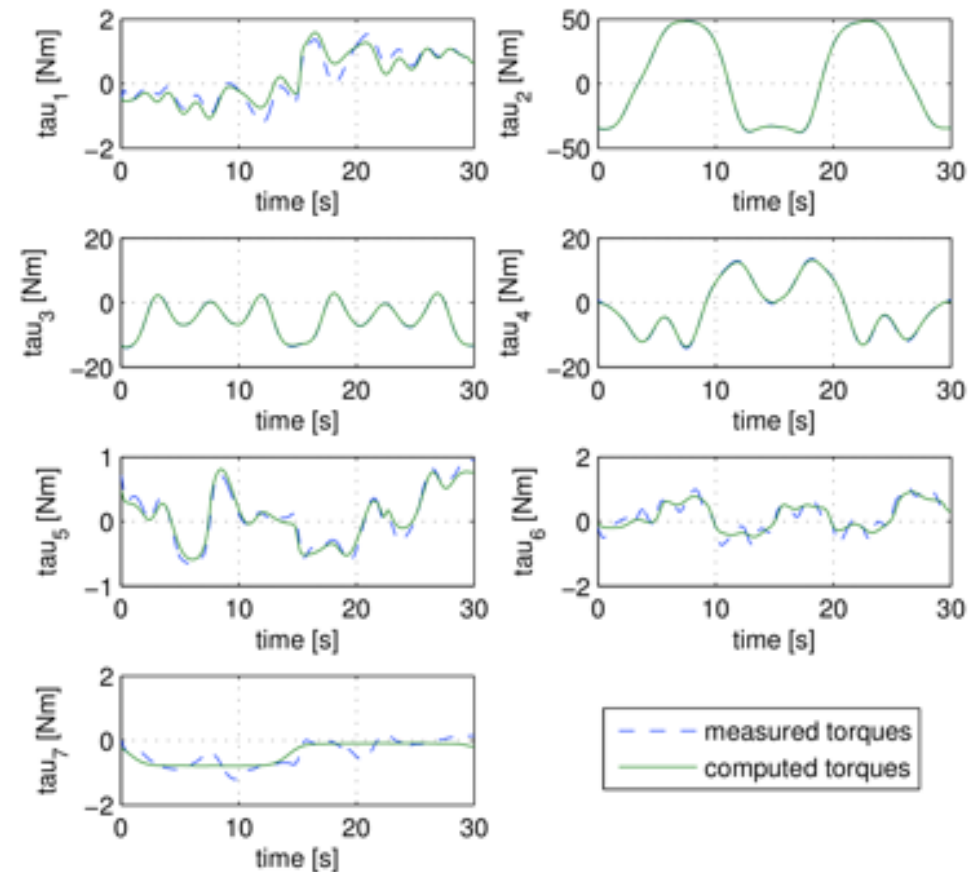
video



data acquisition for identification

dynamic coefficients: 30 inertial, 12 for gravity

C. Gaz, F. Flacco, A. De Luca:
"Identifying the dynamic model used by the KUKA LWR:
A reverse engineering approach", IEEE ICRA 2014



validation after identification (for all 7 joints):
on new desired trajectories, compare
torques computed with the identified model
and torques measured by joint torque sensors



Identification of LWR4 gravity terms

using the linear parametrization, gravity terms can also be identified **separately**

$$\pi_g = \begin{pmatrix} c_{7y}m_7 \\ c_{7x}m_7 \\ c_{6x}m_6 \\ c_{6z}m_6 + c_{7z}m_7 \\ c_{5z}m_5 - c_{6y}m_6 \\ c_{5x}m_5 \\ c_{5y}m_5 + c_{4z}m_4 + d_2(m_5 + m_6 + m_7) \\ c_{4x}m_4 \\ c_{4y}m_4 + c_{3z}m_3 \\ c_{2x}m_2 \\ c_{3x}m_3 \\ c_{2z}m_2 - c_{3y}m_3 + d_1(m_3 + m_4 + m_5 + m_6 + m_7) \end{pmatrix}$$

$$g(q) = Y_g(q)\pi_g$$

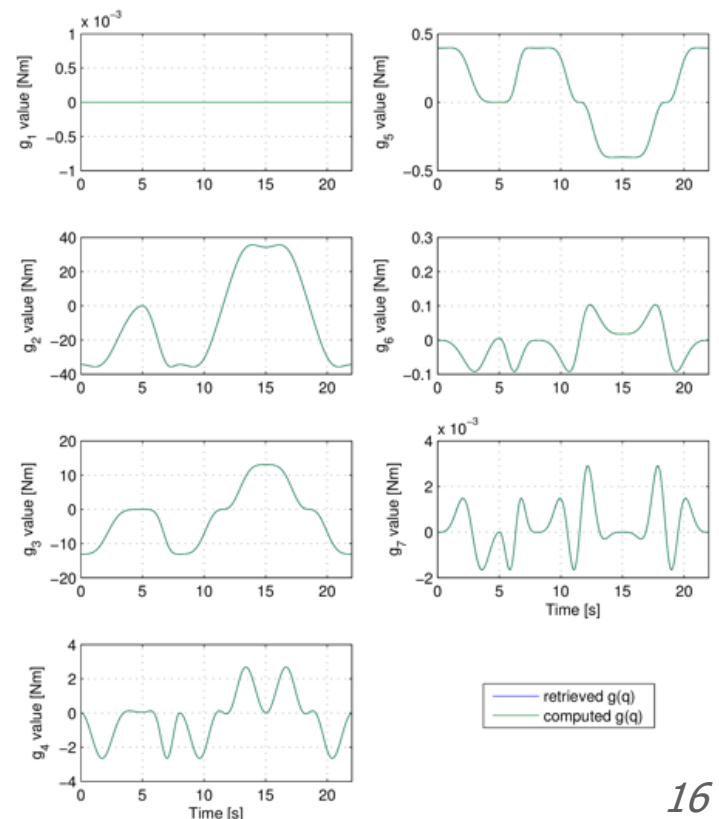
symbolic expressions of gravity-related dynamic coefficients

some small values may also be discarded ...

$$\hat{\pi}_g = \begin{pmatrix} 9.5457 \times 10^{-4} \\ -2.9826 \times 10^{-4} \\ 8.3524 \times 10^{-4} \\ 0.0286 \\ -0.0407 \\ -6.5637 \times 10^{-4} \\ 1.334 \\ -0.0035 \\ -4.7258 \times 10^{-4} \\ 0.0014 \\ 9.4532 \times 10^{-4} \\ 3.4568 \end{pmatrix}$$

numerical values identified through experiments

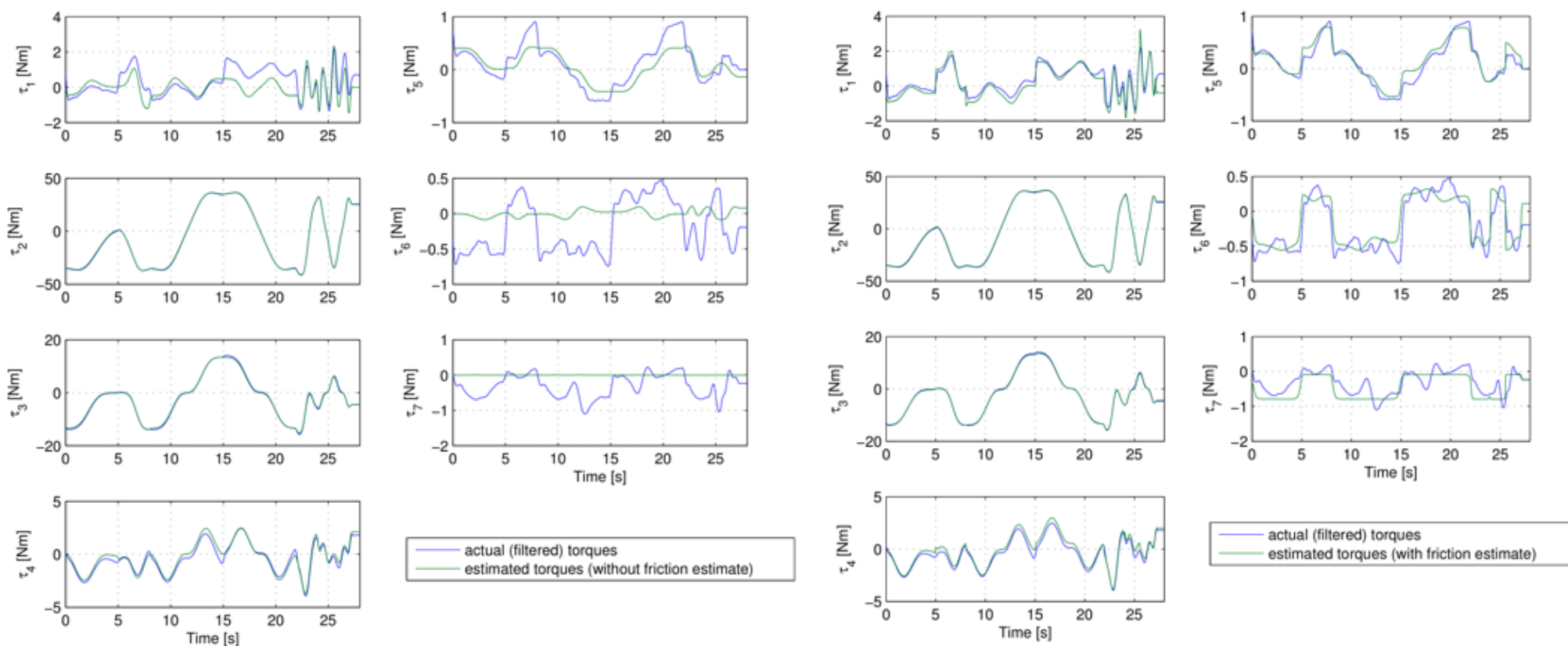
gravity joint torques prediction/evaluation on new validation trajectory



Role of friction in identification

KUKA LWR4 dynamic model estimation vs. joint torque sensor measurement

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = \tau - \tau_{friction} = \tau_{meas}$$

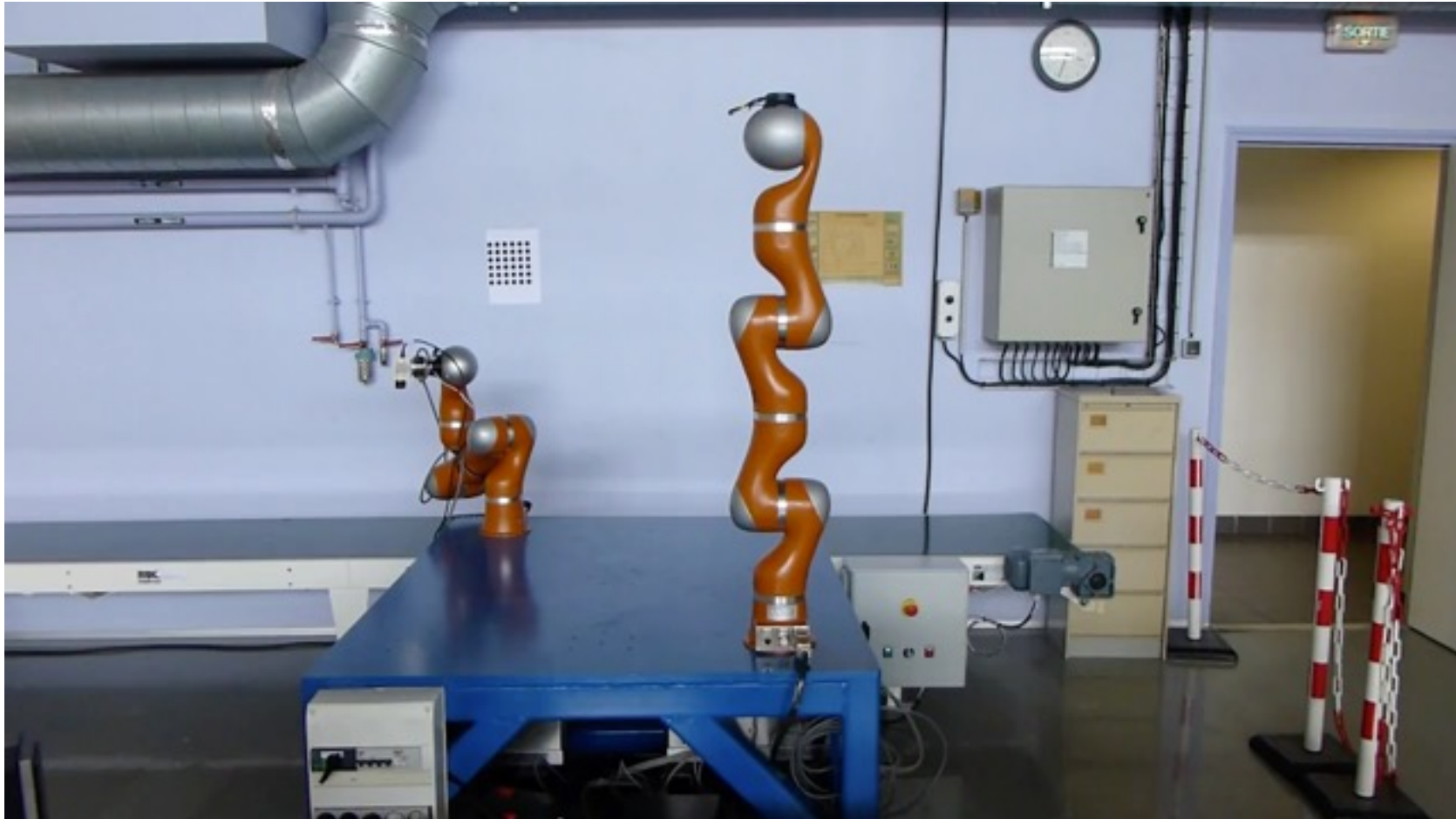


without the use of a joint friction model

including an identified joint friction model

$$\tau_{f,j}(\dot{q}_j) = \frac{\varphi_{1,j}}{1 + e^{-\varphi_{2,j}(\dot{q}_j + \varphi_{3,j})}} - \frac{\varphi_{1,j}}{1 + e^{-\varphi_{2,j}\varphi_{3,j}}}$$

Dynamic identification of KUKA LWR4



video

using more dynamic robot motions for model identification

J. Hollerbach, W. Khalil, M. Gautier: "Ch. 6: Model Identification", Springer Handbook of Robotics (2nd Ed), 2016
free access to multimedia extension: <http://handbookofrobotics.org>



Adding a payload to the robot

- in several industrial applications, changes in the robot payload are often needed
 - using different tools for various technological operations such as polishing, welding, grinding, ...
 - pick-and-place tasks of objects having unknown mass
- what is the rule of change for dynamic parameters when there is an additional payload?
 - do we obtain again a linearly parameterized problem?
 - does this property rely on some specific choice of reference frames (e.g., conventional or modified D-H)?



Rule of change in dynamic parameters

- only the dynamic parameters of the link where a load is added will change (typically, added to the last one –link n – as payload)
 - last link dynamic parameters: m_n (mass), $\mathbf{c}_n = (c_{nx} \ c_{ny} \ c_{nz})^T$ (center of mass), \mathbf{I}_n (inertia tensor expressed **w.r.t. frame n**)
 - payload dynamic parameters: m_L (mass), $\mathbf{c}_L = (c_{Lx} \ c_{Ly} \ c_{Lz})^T$ (center of mass), \mathbf{I}_L (inertia tensor expressed **w.r.t. frame n**)

- mass

$$m_n \rightarrow m_n + m_L$$

- center of mass

$$c_{ni}m_n \rightarrow \frac{c_{ni}m_n + c_{Li}m_L}{m_n + m_L} (m_n + m_L) = c_{ni}m_n + c_{Li}m_L$$

(weighted average)

where $i = x, y, z$

- inertia tensor

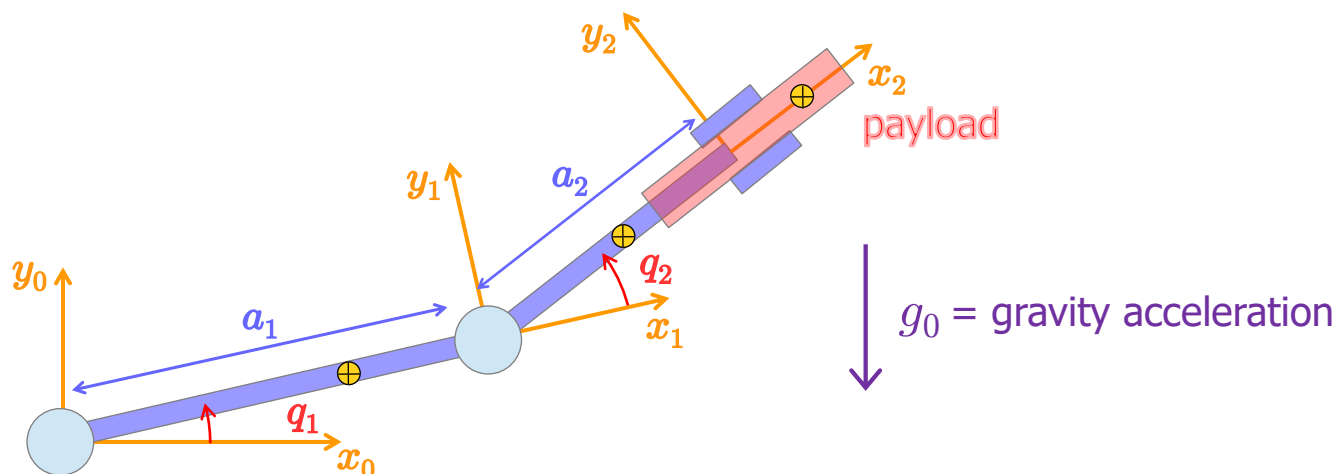
$$\mathbf{I}_n \rightarrow \mathbf{I}_n + \mathbf{I}_L$$

valid **only if** tensors are expressed w.r.t. the **same** reference frame (i.e., frame n)!

- **linear** parametrization is preserved with any kinematic convention (the parameters of the last link will always appear in the form shown above)



Example: 2R planar robot with payload



unloaded robot dynamics $Y\pi = \tau$

$$\pi = \begin{pmatrix} \frac{1}{2} (m_2 a_2^2 + I_{2zz}) + a_2 c_{2x} m_2 \\ c_{2x} m_2 + a_2 m_2 \\ c_{2y} m_2 \\ \frac{1}{2} (I_{1zz} + a_1^2 m_1 + a_1^2 m_2) + a_1 c_{1x} m_1 \\ c_{1x} m_1 + a_1 m_1 + a_1 m_2 \\ c_{1y} m_1 \end{pmatrix}$$



loaded robot dynamics $Y\pi^L = \tau^L$

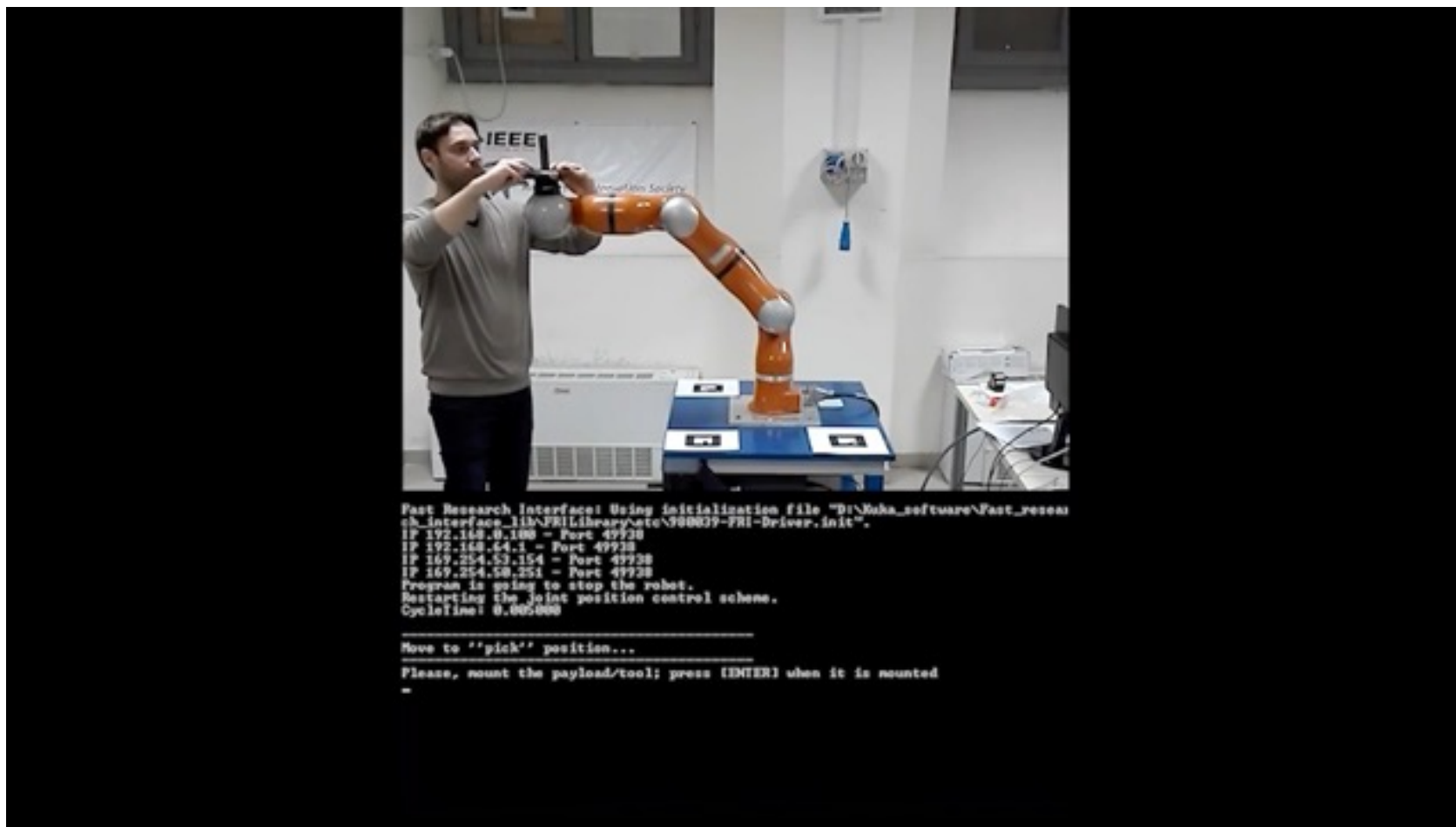
$$\pi^L = \begin{pmatrix} \frac{1}{2} (a_2^2 (m_2 + m_L) + I_{2zz} + I_{Lzz}) + a_2 (c_{2x} m_2 + c_{Lx} m_L) \\ c_{2x} m_2 + c_{Lx} m_L + a_2 (m_2 + m_L) \\ c_{2y} m_2 + c_{Ly} m_L \\ \frac{1}{2} (I_{1zz} + a_1^2 m_1 + a_1^2 (m_2 + m_L)) + a_1 c_{1x} m_1 \\ c_{1x} m_1 + a_1 m_1 + a_1 (m_2 + m_L) \\ c_{1y} m_1 \end{pmatrix}$$

Note 1: position of the center of mass of the two links and of the payload may also be **asymmetric**

Note 2: link inertia & center of mass are expressed in the **DH kinematic frame** attached to the link (e.g., I_{2zz} is the inertia of the second link around the axis z_2)



Validation on the KUKA LWR4 robot



video

C. Gaz, A. De Luca: "Payload estimation based on identified coefficients of robot dynamics – with an application to collision detection" IEEE IROS 2017, Vancouver, September 2017



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KUKA
LWR4 (7R)



Universal Robot
UR10 (6R)



Franka Emika
Panda (7R)

