Robotics 2

Kinematic calibration

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Direct kinematics

- **nominal** set of Denavit-Hartenberg (D-H) parameters
  
  \[
  \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}, \quad 
  a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}, \quad 
  d = \begin{pmatrix} d_1 \\ \vdots \\ d_n \end{pmatrix}
  \]

- **nominal** direct kinematics

  \[
  r_{\text{nom}} = f(\alpha, a, d, \theta)
  \]

  \(\theta\) are typically measured by encoders

  for simplicity, suppose an all-revolute joints manipulator
Need for calibration

- **tolerances** in mechanical construction and in **assembly** of links/joints imply **small** errors in actual end-effector pose (real ≠ nominal parameters)
- **encoder mounting** on motor axes may not be consistent with the “zero reference” of the robot direct kinematics (joint angle measures are constantly biased)
- errors distributed “along” the arm are **amplified**, due to the open chain kinematic structure of most robots
- **calibration goal**: recover as much as possible E-E pose errors by correcting the nominal set of D-H parameters, based on independent external (accurate!) measurements
- **experiments** to be done once for each robot, before starting operation… (and maybe repeated from time to time)
Calibration table
Cartesian measurement systems - 2

laser/camera system + triangulation
Cartesian measurement systems - 3

FANUC 6R robot M-710iC/50

3 SMRs (Spherically-Mounted Reflectors)

API laser tracker III
www.apisensor.com

laser tracker + targets on end-effector
Linearization of direct kinematics

\[ \Delta r = r - r_{\text{nom}} = \frac{\partial f}{\partial \alpha} \cdot \Delta \alpha + \frac{\partial f}{\partial a} \cdot \Delta a + \frac{\partial f}{\partial d} \cdot \Delta d + \frac{\partial f}{\partial \theta} \cdot \Delta \theta \]

- Partial Jacobians evaluated in nominal conditions
- "Small" errors obtained by external measurement system
- First-order variations
- Cameras + triangulation
- Calibration table
Calibration equation

\[
\Delta \varphi = \begin{pmatrix}
\Delta \alpha \\
\Delta a \\
\Delta d \\
\Delta \theta
\end{pmatrix}
\]

\[
\Phi = \begin{pmatrix}
\frac{\partial f}{\partial \alpha} & \frac{\partial f}{\partial a} & \frac{\partial f}{\partial d} & \frac{\partial f}{\partial \theta}
\end{pmatrix}
\]

\[
\Delta r = \Phi \cdot \Delta \varphi
\]

\[
\Delta \bar{r} = \begin{pmatrix}
\Delta r_1 \\
\Delta r_2 \\
\vdots \\
\Delta r_\ell
\end{pmatrix}
\]

\[
\bar{\Phi} = \begin{pmatrix}
\Phi_1 \\
\Phi_2 \\
\vdots \\
\Phi_\ell
\end{pmatrix}
\]

\[
\Delta \bar{r} = \bar{\Phi} \Delta \varphi
\]

\(\ell\) experiments \((\ell \gg n)\)

measures

unknowns

regressor matrix evaluated at nominal parameters

full column rank (for sufficiently large \(\ell\))
Under- and over-determined systems of linear equations

\[ \mathbf{A} \mathbf{x} = \mathbf{b} \]

If multiple solutions \( x \) exist (having the same minimum error norm), the pseudoinverse provides the one having the minimum norm.
Calibration algorithm

\[ \Delta \tilde{r} = \Phi \Delta \varphi \]

\[ \Delta \varphi = \Phi^\# \Delta \tilde{r} = \left( \Phi^T \Phi \right)^{-1} \Phi^T \cdot \Delta \tilde{r} \]

\[ \varphi_{\text{nom}} + \Delta \varphi = \varphi' \]

\[ \Delta \tilde{r}' = \Phi' \Delta \varphi \]

\[ \frac{1}{2} \left\| \Phi \Delta \varphi - \Delta \tilde{r} \right\|^2 \]

minimizes

… ITERATE!

evaluated with new values \( \varphi' \)

ew set of DH par’s + “bias” on measures of \( \theta \)
Final comments

- an **iterative least squares** method
  - original problem is **nonlinear** in the unknowns, then linearized using first-order Taylor expansion
- it is useful to calibrate **first** and **separately** those quantities that are less accurate (typically, the encoder bias)
  - keeping the remaining ones at their nominal values
- alternative kinematic descriptions can be used
  - more complex than D-H parameters, but leading to a **better numerical conditioning** of the regressor matrix in calibration algorithm
  - one such description uses the POE (Product Of Exponential) formula
- more in general, **6 base parameters** should also be included
  - to locate 0-th robot frame w.r.t. world coordinate frame (of external sensor)
- accurate calibration/estimation of **real parameters** is a general problem in robotics (and beyond...)
  - for **sensors** (e.g., camera calibration)
  - for **models** (identification of dynamic parameters of a manipulator)
Calibration experiment in a research environment

- automatic data acquisition for simultaneous calibration of
  - robot-camera transformation
  - DH parameters of the manipulator
Industrial calibration experiment

FANUC 3D Laser calibration (with iR Vision)

Robot Controller R-30iA

Main CPU board

Camera port

2D Camera

3D Laser vision sensor

iPendant

Monitor on iPendant

Ethernet

PC for setup

video