Robotics I

Remote Exam — October 27, 2020

Exercise 1

Consider the spatial 4-dof robot with RRPR sequence of joints shown in Fig. 1. In the following, use **only** the generalized coordinates $\boldsymbol{q} = (q_1, q_2, q_3, q_4)$ defined therein. Note that these are **not** the joint variables of a Denavit-Hartenberg convention!

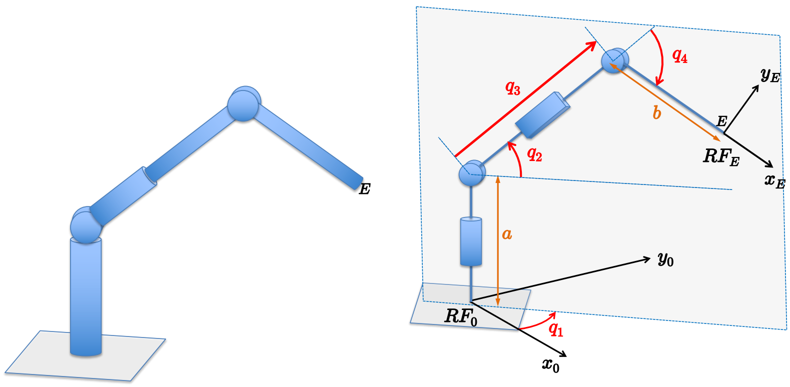

Figure 1: A RRPR robot and its kinematic skeleton, with definition of the joint coordinates $\boldsymbol{q}$.

Exercise 1a

- Determine the direct kinematics, namely the position ${}^0\boldsymbol{p}_E(\boldsymbol{q})$ of the origin and the orientation ${}^0\boldsymbol{R}_E(\boldsymbol{q})$ of the end-effector frame RF_E as functions of the joint variables $\boldsymbol{q}$.

Exercise 1b

- Let $a = 1$ and $b = 0.5$. Assuming that the prismatic joint takes only non-negative values $q_3 \geq 0$, solve the inverse kinematics problem when the (feasible) end-effector pose is given by

$$
{}^0\boldsymbol{A}_E = \begin{pmatrix} 0.5 & 0.5 & \frac{\sqrt{2}}{2} & 0.5 \\ 0.5 & 0.5 & -\frac{\sqrt{2}}{2} & 0.5 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.
$$

Exercise 1c

- Compute the (6×4) geometric Jacobian $\boldsymbol{J}(\boldsymbol{q})$

$$
\begin{pmatrix} \boldsymbol{v}_E \\ \boldsymbol{\omega}_E \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}_L(\boldsymbol{q}) \\ \boldsymbol{J}_A(\boldsymbol{q}) \end{pmatrix} \dot{\boldsymbol{q}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}.
$$

Exercise 1d

- Find all singular configurations of the linear part $\boldsymbol{J}_L(\boldsymbol{q})$ of the geometric Jacobian.

Exercise 1e

- Give the symbolic expression (as a function of the configuration $\boldsymbol{q}$) of a non-trivial joint velocity $\dot{\boldsymbol{q}}_0 \not\equiv \boldsymbol{0}$ such that $\boldsymbol{v}_E = \boldsymbol{J}_L(\boldsymbol{q})\dot{\boldsymbol{q}}_0 = \boldsymbol{0}$ for all possible $\boldsymbol{q}$.

Exercise 2

Consider the motion profile in Fig. 2 for a generic robot joint, parametrized by the amplitude $J > 0$ and the duration $T > 0$. This time profile represents the motion jerk, namely the third time derivative of the joint position $q(t)$, for $t \in [0, T]$.

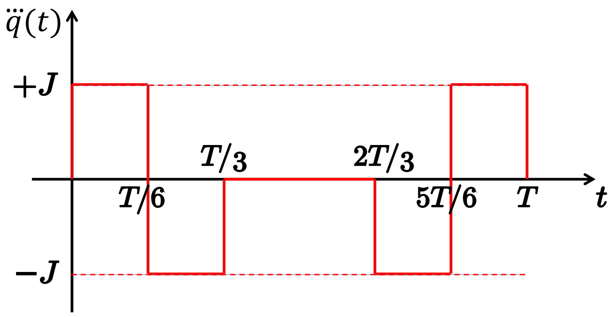

Figure 2: The jerk profile $\dddot{q}(t)$ of the joint motion.

Exercise 2a

- For a (rest-to-rest) motion with zero boundary conditions on velocity and acceleration, determine the value of the net displacement $\Delta = q(T) - q(0)$ as a function of J and T.

Exercise 2b

- Assume now that the initial velocity is $\dot{q}(0) = V > 0$, while $\ddot{q}(0) = 0$ is being kept. What will be then the displacement Δ? Will the final velocity and acceleration be zero at $t = T$?

Exercise 2c

- Assume instead that the initial acceleration is $\ddot{q}(0) = A > 0$, while $\dot{q}(0) = 0$. What will be the displacement Δ in this case? Will the final acceleration be zero at $t = T$?

Exercise 2d

- Let the initial acceleration be $\ddot{q}(0) = A > 0$. What value V should have the initial velocity $\dot{q}(0)$ so that the final velocity $\dot{q}(T)$ is zero? Will the final acceleration be zero at $t = T$?

[180 minutes, open books]