Robotics I

July 15, 2014

For a KUKA LWR robot, let $\boldsymbol{\theta} \in \mathbb{R}^7$ be the joint variables and consider a situation in which the last three joints (constituting a spherical wrist with center $W = O_5 = O_6$) are permanently *frozen*. For kinematic analysis, use the DH frame assignment of Fig. 1, where the robot is shown in its configuration $\boldsymbol{\theta} = \mathbf{0}$. Assume $l_1 = l_2 = l_3 = l_4 = l_5 = l$ (while l_0 and l_6 are different). Frame 7 is drawn for clarity in a displaced position, but is actually located on the final flange of the robot at a distance l_6 from W.

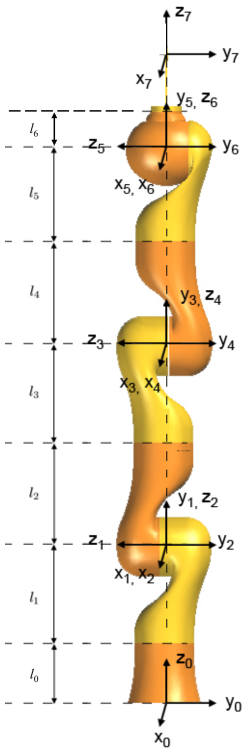

Figure 1: A DH frame assignment for the KUKA LWR robot

- Provide the expression $\boldsymbol{p}_W = \boldsymbol{f}(\boldsymbol{\theta})$ for the position of the robot wrist center W.
- Determine the expression of the 3×4 Jacobian matrix $\boldsymbol{J}(\boldsymbol{\theta})$ relating the velocity of the *active* joints $\dot{\boldsymbol{\theta}}_a \in \mathbb{R}^4$ to the velocity $\boldsymbol{v}_W = \dot{\boldsymbol{p}}_W$.
- Having set $\theta_3 = 0$, find suitable numerical values for the remaining variables in $\boldsymbol{\theta}_a$ so that point W is on the axis $\boldsymbol{z}_0$ at a generic distance d from the origin O_1 of frame 1. The distance d can be chosen arbitrarily, as long as it satisfies $0 < d < 4l$. In the selected configuration, show that the Jacobian $\boldsymbol{J}$ has full rank and give a basis for its null space $\mathcal{N}\{\boldsymbol{J}\}$.
- In the same configuration, show that if also joint 3 is considered to be *frozen*, then the resulting square Jacobian $\boldsymbol{J}_{/3}$ would be singular. Determine then all independent Cartesian directions $\boldsymbol{w}$ that are not instantaneously accessible by the point W (i.e., $\boldsymbol{w} \notin \mathcal{R}\{\boldsymbol{J}_{/3}\}$).

[180 minutes; open books]