Robotics I

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Huge portal robots are used in the aeronautical industry for working on large parts of aircraft bodies. One such robot, having six actuated joints, is shown in Fig. 1. This robot is used for automatic riveting of the plates that constitutes the body of an aircraft.

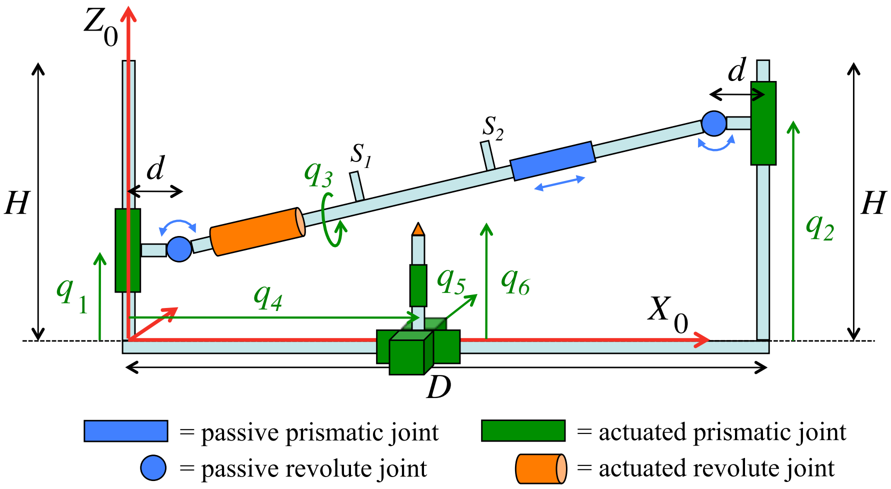

Figure 1: The 6-dof portal robot used for riveting aircraft parts (see the text for definitions and operation)

The portal robot is constituted by two sets of independently actuated axes, the upper section with joints 1 to 3 and the lower section with joints 4 to 6. The upper section carries the part to be worked, while the lower section has the riveting tool on its end effector. In the upper section, two vertical bars of height H are placed at a distance D. Along the two vertical bars, two actuated prismatic joints (with variables $q_1 \in [0, H]$ and $q_2 \in [0, H]$) are used to change the orientation in the vertical plane of a connecting bar (of variable length), which has two supports S_1 and S_2 where the aircraft part will be placed and fixed. The structure contains three passive joints (shown in different colors in Fig. 1), two revolute and one prismatic, that move accordingly to the the actuated prismatic joints. The passive revolute joints, placed at a distance $d << D$ from the vertical bars, transform the linear motions (when different) of q_1 and q_2 in a tilt of the connecting bar (say, by an angle α with respect to the horizontal). The passive prismatic joint accommodates itself so that the connecting bar changes length consistently with the values of q_1 and q_2. Furthermore, the connecting bar can be rotated along its main axis through an actuated revolute joint (with variable $\beta = q_3$). The lower section of the portal robot carries the riveting tool, which can be moved by three actuated prismatic joints with orthogonal axes, horizontally with $q_4 \in [0, D]$ and q_5 (unconstrained in sign), and vertically with $q_6 \in [0, H]$. The riveting tool can operate only along the vertical direction.

Consider now a part of an aircraft body, as shown in Fig. 2. Typically, this is a metallic plate with spatially changing curvature. Each plate is designed at the computer and manufactured with numerically-controlled machines (i.e., through a CAD/CAM system). For this, a reference frame $(\boldsymbol{x}_b, \boldsymbol{y}_b, \boldsymbol{z}_b)$ is attached to the plate for defining its geometry. In order to join two plates together and/or each plate to a stiff supporting structure, a large number of rivets have to be placed in small holes previously drilled on the plate surface (the total number of rivets can go up to three millions for a whole airplane in the Airbus 300 class). The position of a riveting point is specified by vector $\boldsymbol{p}$ and, at this point, the (external) normal to the surface is specified by the unit vector $\boldsymbol{n}$. Both these vectors are expressed in the frame attached to the plate. In order to be successful, the riveting task should be performed as accurately as possible along the normal to the plate surface.

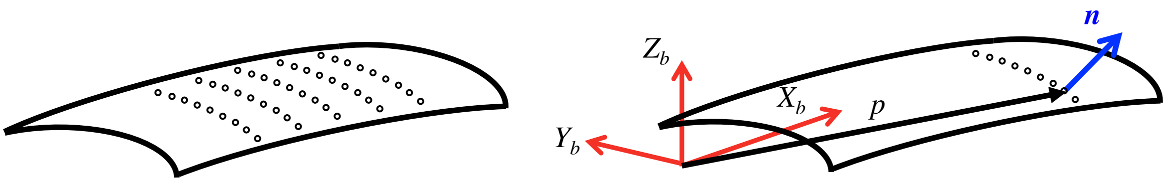

Figure 2: An aircraft plate (left) and its associated reference frame (right): The position vector $\boldsymbol{p}$ locates a riveting point, where the normal to the surface is given by the unit vector $\boldsymbol{n}$

Figure 3 shows the plate mounted on the portal robot (in particular, fixed to the supports S_1 and S_2 of the connecting bar in the upper section of the robot). The correct mounting is done so that the origin of frame $(\boldsymbol{x}_b, \boldsymbol{y}_b, \boldsymbol{z}_b)$ coincides with the passive revolute joint on the left, and the $\boldsymbol{x}_b$ axis is aligned with the connecting bar (i.e., along the axis of the passive prismatic joint). In correspondence to $q_3 = 0$, the $\boldsymbol{z}_b$ axis is in the vertical plane.

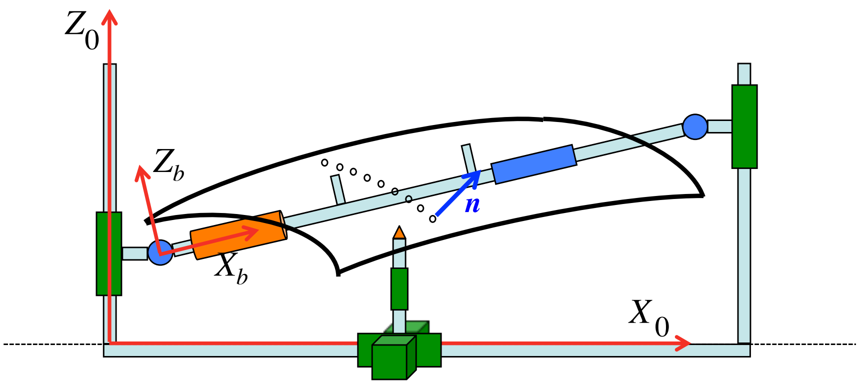

Figure 3: The aircraft plate correctly mounted on the two supports of the connecting bar of the upper section of the portal robot

With the above description in mind, a single riveting task with the portal robot is executed as follows. The upper section of the robot should move the part so that the normal $\boldsymbol{n}$ to the surface at the riveting point is oriented along the upward vertical direction. Then, the end effector carrying the riveting tool on the lower section of the robot is moved so as to position itself at the riveting point.

1. Given the normal ${}^b\boldsymbol{n}$ to the surface plate at the riveting point, find the inverse kinematic solution for the two pointing angles (α, β) in closed symbolic form. Note that two solutions can be found in general, but one may not be feasible because of the limits on the robot joint motions.

2. Each obtained pair (α, β) needs to be realized by suitable $\boldsymbol{q}_u = (q_1, q_2, q_3)$ of the upper section of the portal robot. Since this is an underdetermined problem, an infinite number of solutions exists. Choose a solution to this redundant problem, determining a single value of $\boldsymbol{q}_u$ that allows the pointing task to be realized in the most convenient way.

3. Consider, in addition to ${}^b\boldsymbol{n}$, a given position ${}^b\boldsymbol{p}$ of the riveting point. With the solution found for the pointing subtask, determine the explicit (unique) expression of $\boldsymbol{q}_l = (q_4, q_5, q_6)$ that will position the end-effector riveting tool at the desired point of the plate.

4. Provide a feasible numerical solution $\boldsymbol{q} = (\boldsymbol{q}_u, \boldsymbol{q}_l) = (q_1, q_2, q_3, q_4, q_5, q_6)$, using the robot data
$$D = 10 \text{ [m]}, \qquad H = 5 \text{ [m]}, \qquad d = 0.75 \text{ [m]},$$
for a riveting task specified by
$${}^b\boldsymbol{p} = \begin{pmatrix} 5.5 \\ -0.3 \\ -0.2 \end{pmatrix} \text{ [m]}, \qquad {}^b\boldsymbol{n} = \begin{pmatrix} 0.5 \\ 0.8138 \\ -0.2962 \end{pmatrix}.$$

[Hint]: There is no need of assigning D-H frames/parameters. Work instead with suitable frames.

[240 minutes; open books]

Solution

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The first part of the problem is a pointing task in which the joint variables $\boldsymbol{q}_u = (q_1, q_2, q_3)$ are used to bring the unit normal vector $\boldsymbol{n}$ to the surface aligned with $\boldsymbol{z}_0$. When expressed in the base frame $(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{z}_0)$, this alignment is represented by

$$ {}^0\boldsymbol{n} = {}^0\boldsymbol{R}_b\,{}^b\boldsymbol{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(= {}^0\boldsymbol{z}_0\right), \tag{1} $$

where the rotation matrix ${}^0\boldsymbol{R}_b$ depends on $\boldsymbol{q}_u = (q_1, q_2, q_3)$. To describe this pointing task, it is convenient to introduce the pair of angles (α, β) as

$$ \alpha = \arctan\left(\frac{q_1 - q_2}{D - 2d}\right) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \qquad \beta = q_3. \tag{2} $$

Note that the domain of definition for α is actually contained in the indicated one, but these limits show that we can use directly the arctangent function and not the ATAN2. In particular, due to the limits of joints 1 and 2, we have that

$$ \max|\alpha| = \arctan\left(\frac{H}{D - 2d}\right) > 0. \tag{3} $$

To determine ${}^0\boldsymbol{R}_b$, we can use an intermediate frame $(\boldsymbol{x}_d, \boldsymbol{y}_d, \boldsymbol{z}_d)$, with origin coincident with that of $(\boldsymbol{x}_b, \boldsymbol{y}_b, \boldsymbol{z}_b)$ and rotated by α around the $\boldsymbol{y}_0$ axis. Therefore,

$$ {}^0\boldsymbol{R}_b = {}^0\boldsymbol{R}_d(\alpha)\,{}^d\boldsymbol{R}_b(\beta) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha s_\beta & s_\alpha c_\beta \\ 0 & c_\beta & -s_\beta \\ -s_\alpha & c_\alpha s_\beta & c_\alpha c_\beta \end{pmatrix}, $$

where a compact notation has been used in the last expression. The $\boldsymbol{n}$ axis can be taken as one (actually, any) of the coordinate axes of a frame $(\boldsymbol{x}_n, \boldsymbol{y}_n, \boldsymbol{z}_n)$ attached to the riveting point. We choose for instance the $\boldsymbol{x}_n = \boldsymbol{n}$ axis (i.e., ${}^n\boldsymbol{n} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$), so that

$$ {}^b\boldsymbol{R}_n = \begin{pmatrix} {}^b\boldsymbol{n} & {}^b\boldsymbol{s} & {}^b\boldsymbol{a} \end{pmatrix}, \qquad {}^b\boldsymbol{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}. $$

Our pointing task equation (1), i.e., the direct kinematics of the task, is then

$$ {}^0\boldsymbol{R}_d(\alpha)\,{}^d\boldsymbol{R}_b(\beta)\,{}^b\boldsymbol{R}_n \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} n_x c_\alpha + n_y s_\alpha s_\beta + n_z s_\alpha c_\beta \\ n_y c_\beta - n_z s_\beta \\ -n_x s_\alpha + n_y c_\alpha s_\beta + n_z c_\alpha c_\beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{4} $$

Squaring and adding the first and third equations in (4) gives

$$ n_x^2 + n_y^2 s_\beta^2 + n_z^2 c_\beta^2 + 2 n_y n_z s_\beta c_\beta = n_x^2 + \left(n_y s_\beta + n_z c_\beta\right)^2 = 1 $$

or

$$ n_y s_\beta + n_z c_\beta = \pm\sqrt{1 - n_x^2}. \tag{5} $$

From eq. (5) and the second equation in (4), we obtain a linear system in the s_β and c_β unknowns:

$$\begin{pmatrix} n_y & n_z \\ -n_z & n_y \end{pmatrix} \begin{pmatrix} s_\beta \\ c_\beta \end{pmatrix} = \begin{pmatrix} \pm\sqrt{1-n_x^2} \\ 0 \end{pmatrix}. \tag{6}$$

This system can be solved uniquely, provided that the determinant of the coefficient matrix is not zero. Singularity occurs if and only if

$$n_y^2 + n_z^2 = 0 \qquad \Longleftrightarrow \qquad n_y = n_z = 0,\ n_x = \pm 1.$$

It is easy to see from eq. (4) that this happens for $\alpha = -\pi/2$ (if $n_x = 1$) or for $\alpha = \pi/2$ (if $n_x = -1$), with β being undefined in both cases. However, this situation is unfeasible (and never encountered in practice) for the upper section of the portal robot. For $n_y^2 + n_z^2 \neq 0$, solving (6) yields

$$s_\beta = \frac{\pm\, n_y}{n_y^2 + n_z^2}\sqrt{1-n_x^2} \qquad\qquad c_\beta = \frac{\pm\, n_z}{n_y^2 + n_z^2}\sqrt{1-n_x^2} \tag{7}$$

and thus (eliminating common positive terms)

$$\beta = \text{ATAN2}\,\{\pm n_y, \pm n_z\}, \tag{8}$$

which are the two possible solutions (depending on the upper or lower signs chosen in the arguments of ATAN2). Replacing (5) in the first and third equations of (4), we can eliminate the appearance of β and obtain a linear system in the s_α and c_α unknowns

$$\begin{pmatrix} \pm\sqrt{1-n_x^2} & n_x \\ -n_x & \pm\sqrt{1-n_x^2} \end{pmatrix} \begin{pmatrix} s_\alpha \\ c_\alpha \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{9}$$

with nonsingular coefficient matrix (its determinant is 1). Its solution is

$$s_\alpha = -n_x, \qquad c_\alpha = \pm\sqrt{1-n_x^2} \tag{10}$$

and thus

$$\alpha = \text{ATAN2}\left\{-n_x, \pm\sqrt{1-n_x^2}\right\}, \tag{11}$$

with the upper or lower sign chosen in correspondence to the two solutions for β. As a result, two pairs of solutions (α, β) have been found. The value of α in these pairs should be checked against the feasible limit (3): if this is exceeded, the associated solution pair should be discarded.

For each (α, β) pair, we associate now a suitable unique value of $\boldsymbol{q}_u = (q_1, q_2, q_3)$ by resolving the intrinsic redundancy in the following convenient way. Consider a value $h > 0$ for the average of the two prismatic joints q_1 and q_2, i.e.,

$$\frac{q_1 + q_2}{2} = h.$$

Putting this together with the definition (2) of α yields the nonsingular (the determinant is 1) linear system

$$\begin{pmatrix} 1 & -1 \\ 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} (D-2d)\tan\alpha \\ h \end{pmatrix},$$

with the unique solution

$$q_1 = h + \frac{(D-2d)\tan\alpha}{2}, \qquad q_2 = h - \frac{(D-2d)\tan\alpha}{2}. \tag{12}$$

It is immediate to see that the value $h = H/2$ guarantees the best use of the available joint ranges $[0, H]$ for q_1 and q_2. Finally, for the third joint variable we set simply

$$q_3 = \beta. \tag{13}$$

As a result, two inverse kinematic solutions for the complete vector $\boldsymbol{q}_u = (q_1, q_2, q_3)$.

Consider now the problem of positioning the riveting tool on the lower section of the robot at the given position ${}^b\boldsymbol{p} = \begin{pmatrix} p_x & p_y & p_z \end{pmatrix}^T$, a vector starting from the origin of the frame $(\boldsymbol{x}_b, \boldsymbol{y}_b, \boldsymbol{z}_b)$ (see Fig. 2) and expressed in this frame. Given one of the pairs (α, β) found in the previous pointing subtask, we compute the following positional direct kinematics

$$ {}^0\boldsymbol{p} = \begin{pmatrix} d \\ 0 \\ q_1 \end{pmatrix} + {}^0\boldsymbol{R}_d(\alpha)\,{}^d\boldsymbol{R}_b(\beta)\,{}^b\boldsymbol{p}. \tag{14}$$

Since the lower section of the portal robot is a Cartesian structure, the (unique) inverse kinematics solution is given simply by

$$\boldsymbol{q}_l = \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix} = {}^0\boldsymbol{p} = \begin{pmatrix} d + p_x c_\alpha + s_\alpha \left(p_y s_\beta + p_z c_\beta\right) \\ p_y c_\beta - p_z s_\beta \\ q_1 - p_x s_\alpha + c_\alpha \left(p_y s_\beta + p_z c_\beta\right) \end{pmatrix}. \tag{15}$$

We evaluate the formulas on the provided numerical data. From eq. (11) we obtain the two values

$$\alpha_1 = -0.5236\,[\text{rad}] = -30^\circ, \qquad \alpha_2 = -2.6180\,[\text{rad}] = -150^\circ,$$

while from eq. (8) we have

$$\beta_1 = 1.9199\,[\text{rad}] = 110^\circ, \qquad \beta_2 = -1.2217\,[\text{rad}] = -70^\circ.$$

From eq. (3), the maximum absolute value of α is 0.5317 [rad]$= 30.4655^\circ$. Therefore, the solution pair (α_2, β_2) is unfeasible and should be discarded. From the pair (α_1, β_1), using eq. (12) with $h = H/2 = 2.5$ and eq. (13), we find

$$q_1 = 0.0463\,[\text{m}], \qquad q_2 = 4.9537\,[\text{m}], \qquad q_3 = 1.9199\,[\text{rad}].$$

Finally, from eq. (15) we obtain

$$q_4 = 5.6199\,[\text{m}], \qquad q_5 = 0.2905\,[\text{m}], \qquad q_6 = 2.6114\,[\text{m}].$$

All joint variables are in their admissible range. A Matlab code is available for this numerical evaluation.

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