Robotics I – Homework 1

Hand out: November 15, 2011; Return on: November 22, 2011

Figures 1 and 2 show different views of the Comau Smart5 NJ4 170 robot for spot welding. This is a 6R anthropomorphic manipulator with a fully symmetric geometry, having a parallelogram structure for motion transmission (for links 2 and 3), and a so-called *hollow wrist* that allows to keep welding gun service and power cables all inside the robot. This wrist is non-spherical, with the last three joint axes intersecting two by two.

Figure 1: Rear and frontal CAD views of the Comau NJ4 robot

Figure 2: Perspective and side views of the Comau NJ4 robot

In the following analysis, neglect the presence of the parallelogram structure (a closed kinematic chain).

1. Assign the robot frames according to the Denavit-Hartenberg convention and provide the associated table of parameters. Use symbols for numerically unknown quantities. Note that the twist angles of joint axes 5 and 6 can be chosen as equal and opposite (their absolute value, which is $60°$, should be left in symbolic form).

2. Compute (possibly with the help of a symbolic algebraic and manipulation program) the direct kinematics in the form

$$
{}^0\boldsymbol{T}_6(\boldsymbol{q}) = \begin{pmatrix} {}^0\boldsymbol{R}_6(\boldsymbol{q}) & {}^0\boldsymbol{p}_6(\boldsymbol{q}) \\ \boldsymbol{0}^T & 1 \end{pmatrix} = \begin{pmatrix} {}^0\boldsymbol{n}_6(\boldsymbol{q}) & {}^0\boldsymbol{s}_6(\boldsymbol{q}) & {}^0\boldsymbol{a}_6(\boldsymbol{q}) & {}^0\boldsymbol{p}_6(\boldsymbol{q}) \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

where $\boldsymbol{q} = \begin{pmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{pmatrix}^T$.

3. Draw a sketch of the robot in the *zero configuration* ($\boldsymbol{q} = \boldsymbol{0}$), as it follows from the chosen frame assignment. In this configuration, give the expression of the position of origin O_6 of the end-effector frame and of the position of intersection point H between joint axes 4 and 5.

[estimated time: 210 minutes (open books)]

Comau Smart5 NJ4 170

DH frame assignment, parameter table, direct kinematics, and zero configuration

Comau Smart5 NJ4 170
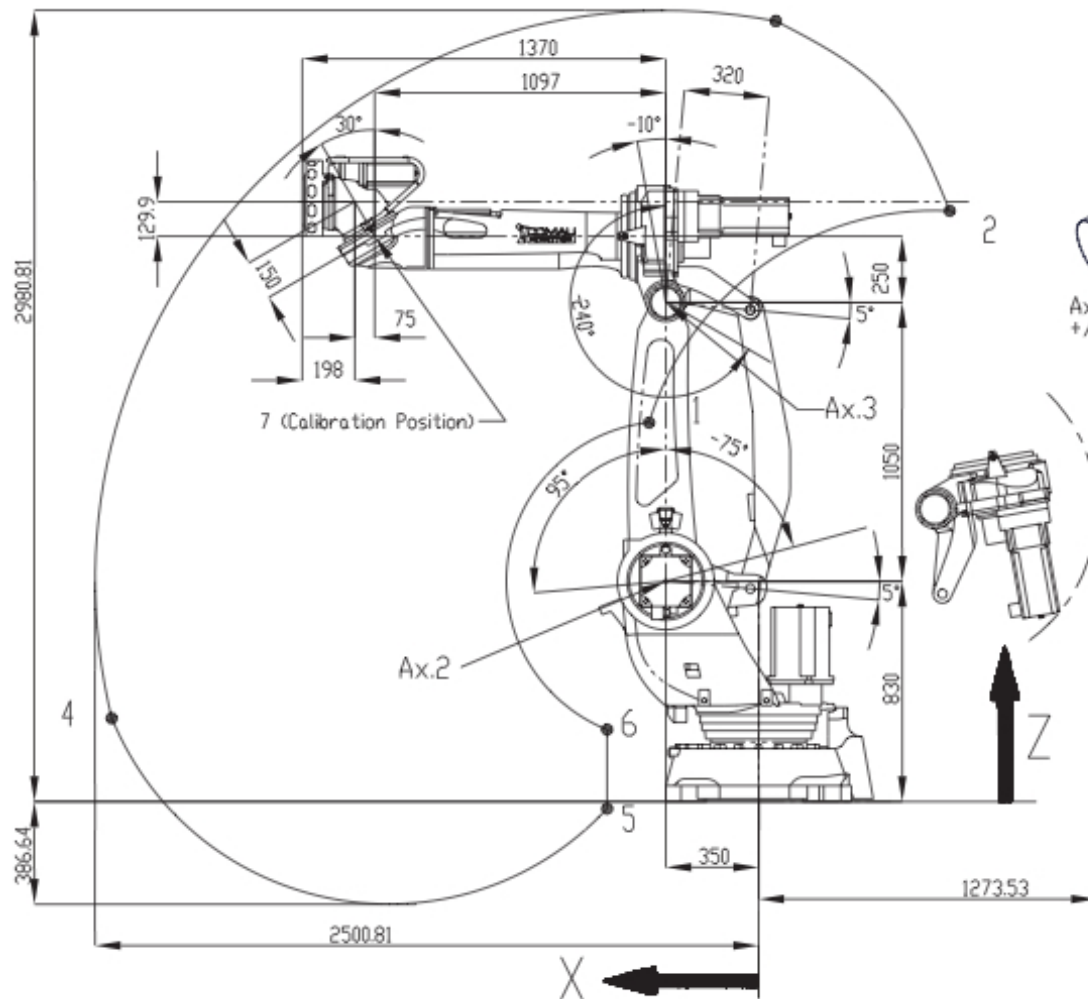
Comau Smart5 NJ4 170

The Spot Welding Reference

The third generation of "hollow wrist" robot family signed Comau Robotics: the new spot welding reference. Ten years experience in designing hollow wrist robots together with the new parallelogram structure of latest generation of industrial robots: SMART5 NJ4. The structure of the robot NJ4 "hollow wrist" makes it possible to keep all the welding gun service and power cables inside the robot. It provides agility, penetrability and applicability characteristics that, at present, cannot be found elsewhere on the market.

Technical Features

- integrated dressing from the base to the flange of the robot
- parallelogram structure with no balancing mass
- perfect symmetry of the mechanical structure
- strong, agile and streamlined kinetic structure
- low power consumption: energy saving, up to 25% more
- possibility of installation both on floor and at ceiling
- AC brushless motors with incremental encoders
- braking system (removable axis by axis)

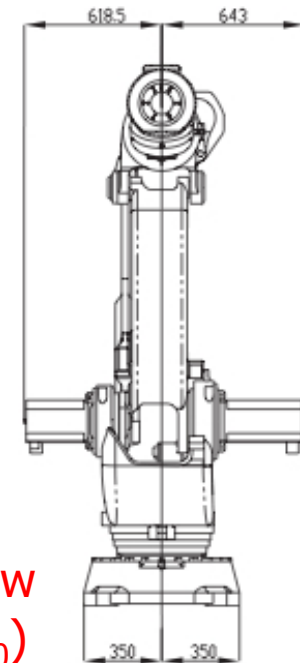**NJ4 170 - 2.5**: maximum horizontal reach 2500 mm, repeatability 0.10 mm
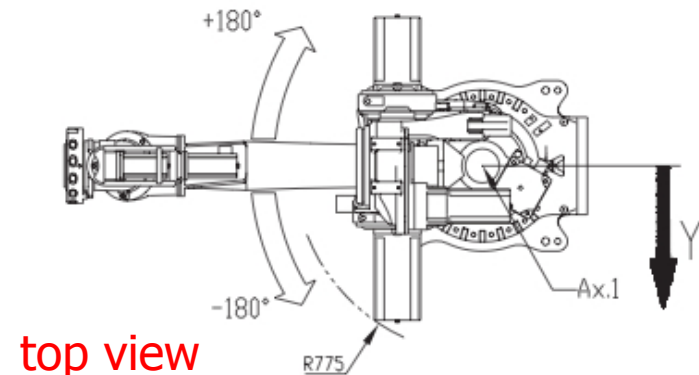
Comau Smart5 NJ4 170
workspace
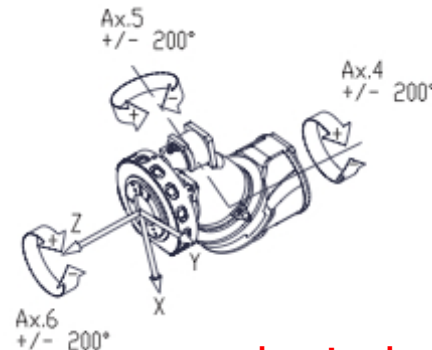
non-spherical wrist (axes 4-6)

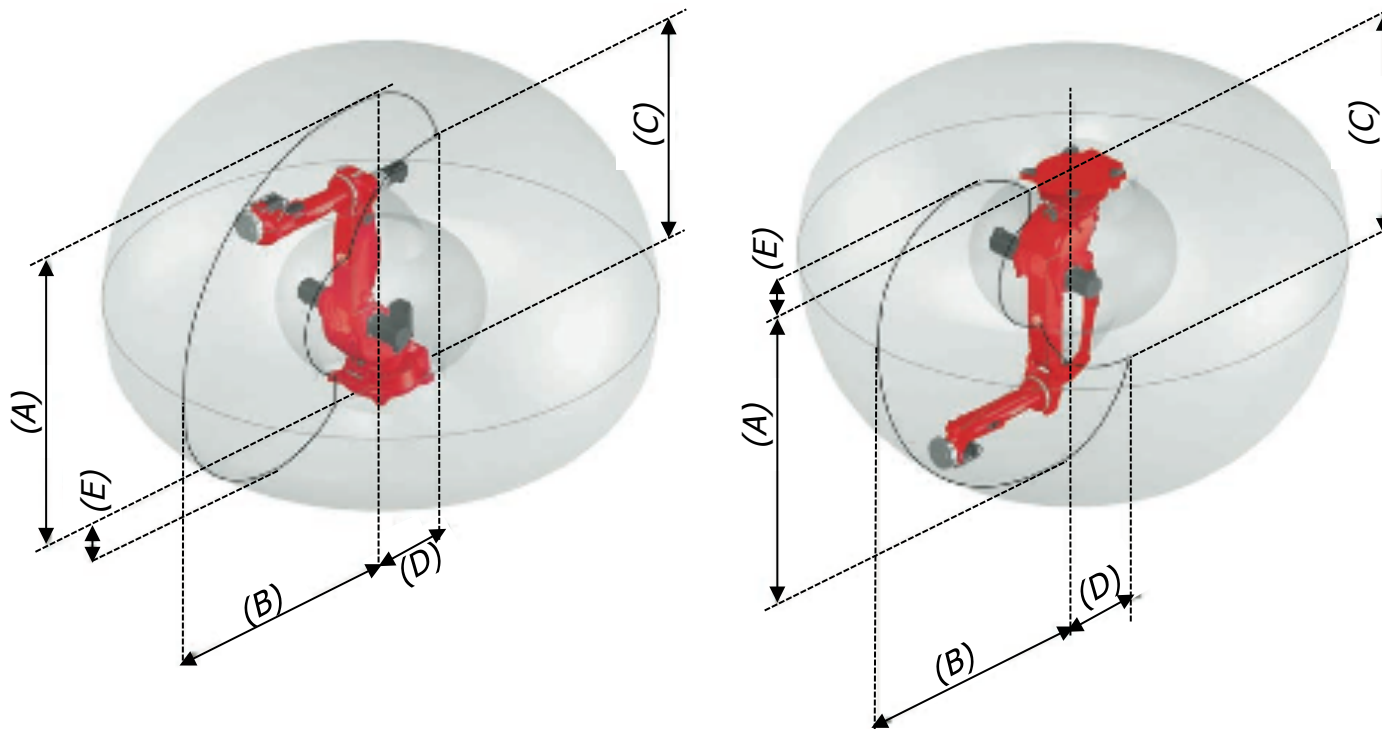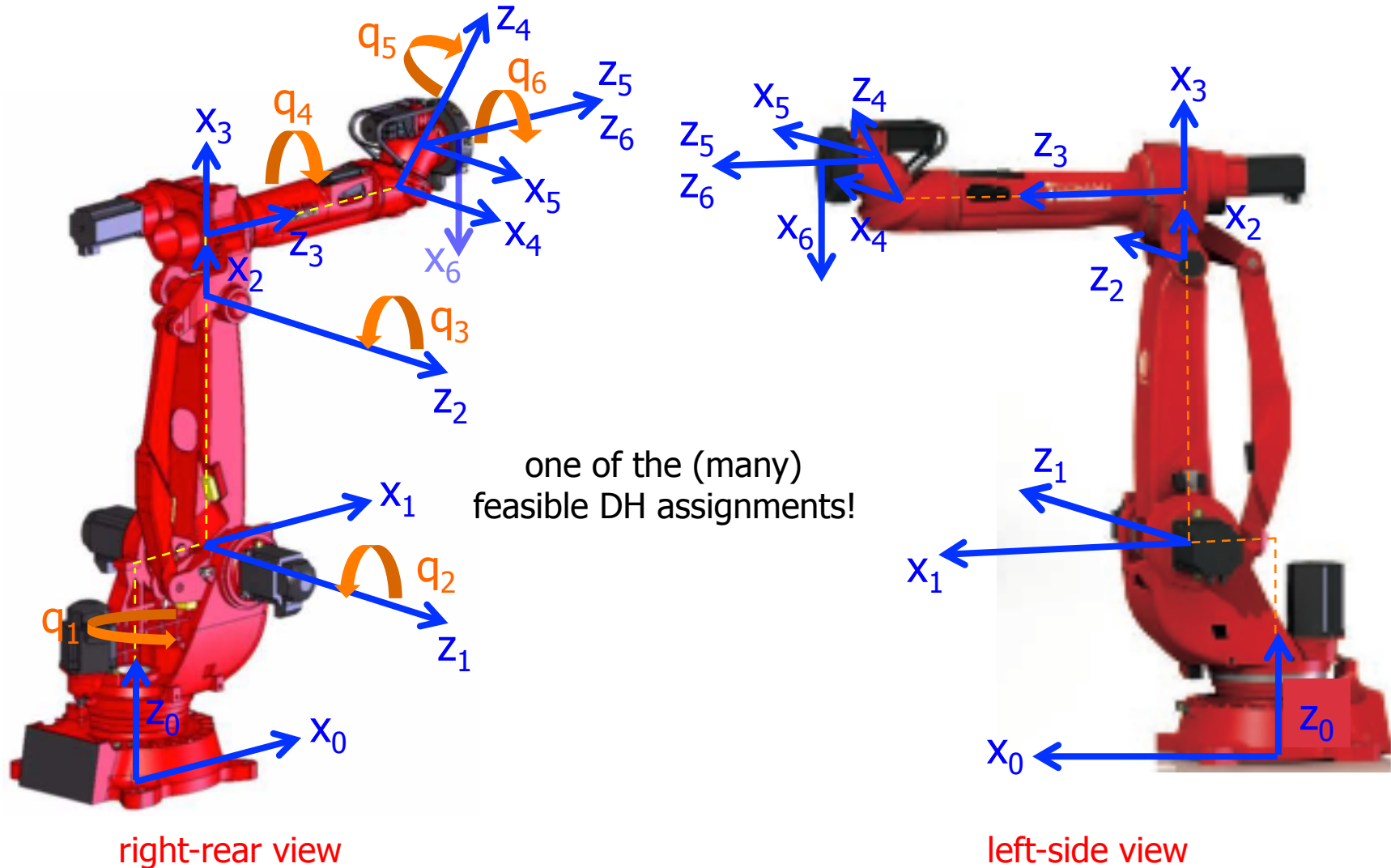front view (from X_0)

left-side view

top view

Comau Smart5 NJ4 170
envelope

A	B	C	D	E	
2981	2501	2226	720	327	[mm]

Note: these data are NOT related to the DH parameters of the following table!
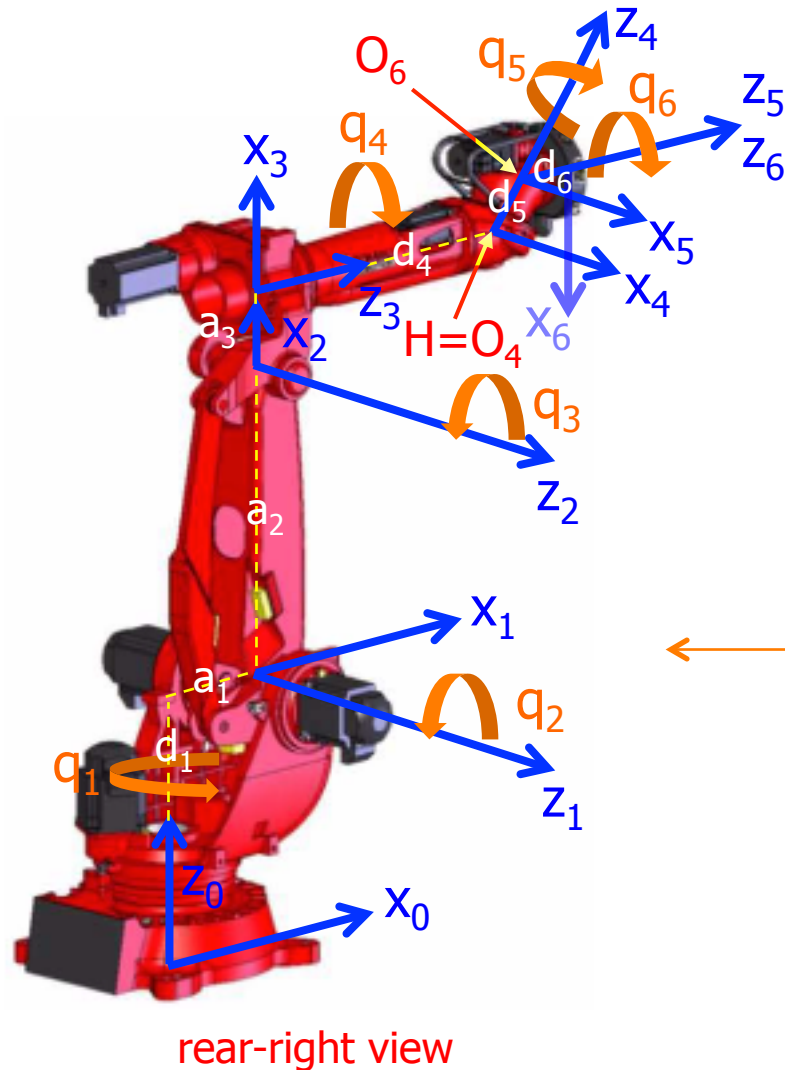
Comau Smart5 NJ4 170
DH frames

one of the (many)
feasible DH assignments!

right-rear view

left-side view

Comau Smart5 NJ4 170
DH table of parameters

rear-right view

i	α_i	a_i	d_i	θ_i
1	$\pi/2$	$a_1 > 0$	$d_1 > 0$	$q_1 = 0$
2	0	$a_2 > 0$	0	$q_2 = \pi/2$
3	$\pi/2$	$a_3 > 0$	0	$q_3 = 0$
4	$\alpha_4 = \alpha > 0$	0	$d_4 > 0$	$q_4 = \pi/2$
5	$\alpha_5 = -\alpha < 0$	0	$d_5 > 0$	$q_5 = 0$
6	0	0	$d_6 > 0$	$q_6 = \pi/2$

in the configuration shown

point H (origin of frame 4)

$$\begin{bmatrix} {}^0 p_4 \\ 1 \end{bmatrix} = {}^0 A_1(q_1)\, {}^1 A_2(q_2)\, {}^2 A_3(q_3)\, {}^3 A_4(q_4) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
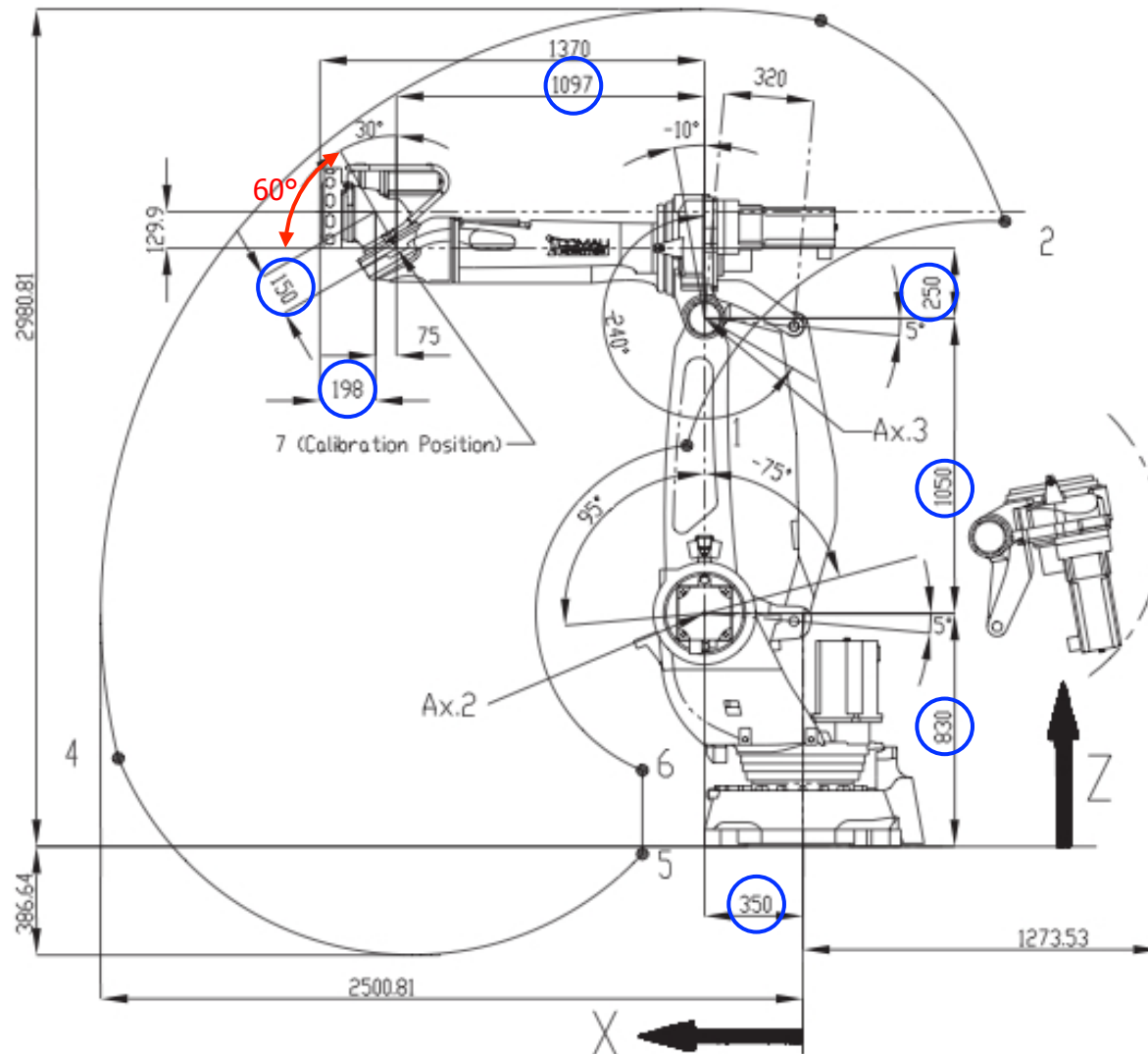
point O_6 (origin of frame 6)

$$\begin{bmatrix} {}^0 p_6 \\ 1 \end{bmatrix} = {}^0 A_1(q_1)\, {}^1 A_2(q_2)\, {}^2 A_3(q_3)\, {}^3 A_4(q_4)\, {}^4 A_5(q_5)\, {}^5 A_6(q_6) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

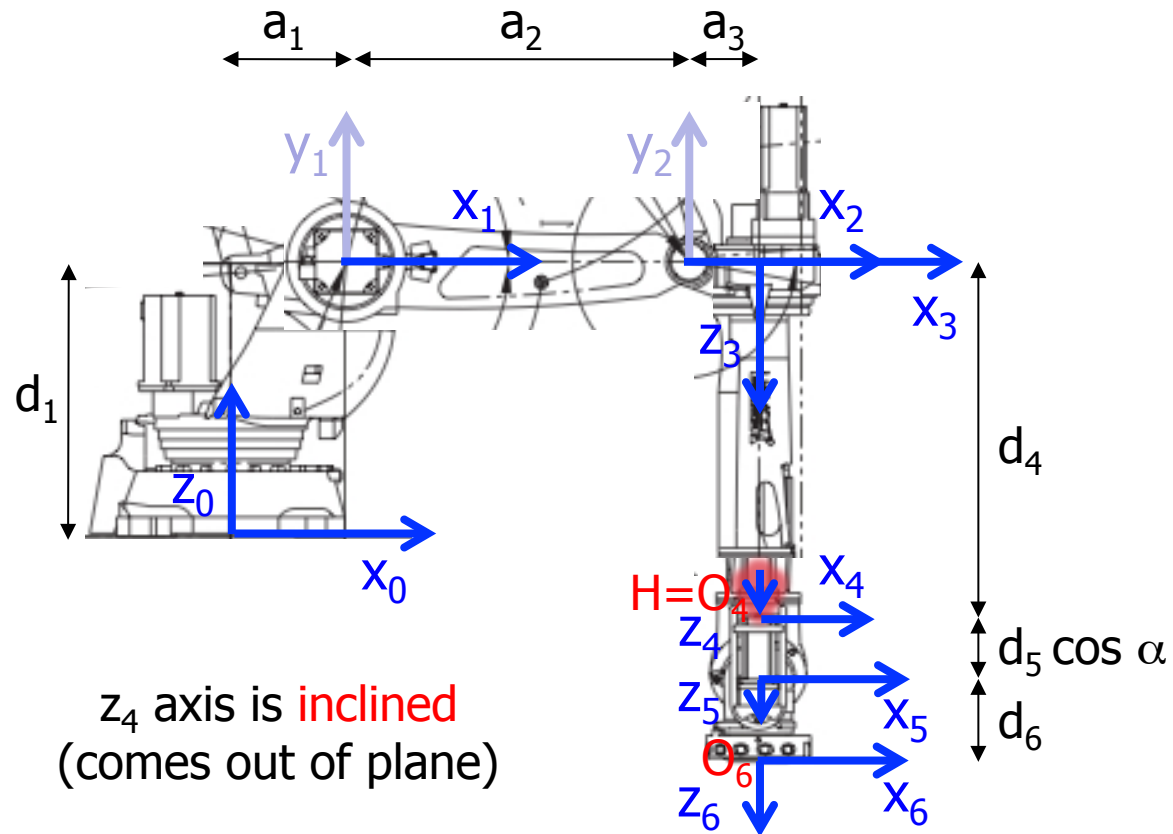# Comau Smart5 NJ4 170
mapping numerical values for the DH parameters

	[mm]
a_1	350
a_2	1050
a_3	250
d_1	830
d_4	1097
d_5	150
d_6	198
α	60°

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DH zero configuration

$q = \theta = 0$

z_4 axis is inclined (comes out of plane)

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DH homogenous transformation matrices

from Matlab (symbolic) program

```
A1 =                                        A2 =

[ cos(q1), 0,  sin(q1), a1*cos(q1)]         [ cos(q2), -sin(q2), 0, a2*cos(q2)]
[ sin(q1), 0, -cos(q1), a1*sin(q1)]         [ sin(q2),  cos(q2), 0, a2*sin(q2)]
[      0, 1,        0,          d1]         [       0,        0, 1,          0]
[      0, 0,        0,           1]         [       0,        0, 0,          1]


A3 =                                        A4 =

[ cos(q3), 0,  sin(q3), a3*cos(q3)] [ cos(q4), -cos(alf)*sin(q4),  sin(alf)*sin(q4),  0]
[ sin(q3), 0, -cos(q3), a3*sin(q3)] [ sin(q4),  cos(alf)*cos(q4), -cos(q4)*sin(alf),  0]
[      0, 1,        0,          0] [       0,          sin(alf),          cos(alf), d4]
[      0, 0,        0,          1] [       0,                 0,                 0,  1]


A5 =                                        A6 =

[ cos(q5), -cos(alf)*sin(q5), -sin(alf)*sin(q5),  0]    [ cos(q6), -sin(q6), 0,  0]
[ sin(q5),  cos(alf)*cos(q5),  cos(q5)*sin(alf),  0]    [ sin(q6),  cos(q6), 0,  0]
[      0,          -sin(alf),          cos(alf), d5]    [       0,        0, 1, d6]
[      0,                  0,                 0,  1]    [       0,        0, 0,  1]
```

Comau Smart5 NJ4 170
on its direct kinematics

the symbolic output for the whole

$$^0T_6(\boldsymbol{q}) = \,^0A_1(q_1)\,^1A_2(q_2)\,^2A_3(q_3)\,^3A_4(q_4)\,^4A_5(q_5)\,^5A_6(q_6)$$

is quite long...

for instance, the symbolic output for the first component of $^0p_6(q)$ is

```
cos(q1)*(al + a3*cos(q2 + q3) + a2*cos(q2))

- sin(alf)*sin(q1)*(d5*cos(q4) + d6*cos(alf)*cos(q4)

+ d6*sin(q4)*sin(q5) - d6*cos(alf)*cos(q4)*cos(q5)

+ sin(q2 + q3)*cos(q1)*(d4 + d5*cos(alf)

+ d6*(cos(alf)^2 - cos(q5)*(cos(alf)^2 - 1)))

+ cos(q2 + q3)*cos(q1)*sin(alf)*(d5*sin(q4) + d6*cos(alf)*sin(q4)

- d6*cos(q4)*sin(q5) - d6*cos(alf)*cos(q5)*sin(q4))

+ cos(q2 + q3)*cos(q1)*sin(alf)*(d5*sin(q4) + d6*cos(alf)*sin(q4)

- d6*cos(q4)*sin(q5) - d6*cos(alf)*cos(q5)*sin(q4))
```

the symbolic output for the position of point H is still manageable

```
H =

a1*cos(q1) + d4*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)) + a2*cos(q1)*cos(q2) + a3*cos(q1)*cos(q2)*cos(q3) - a3*cos(q1)*sin(q2)*sin(q3)
d4*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)) + a1*sin(q1) + a2*cos(q2)*sin(q1) + a3*cos(q2)*cos(q3)*sin(q1) - a3*sin(q1)*sin(q2)*sin(q3)
                                 d1 - d4*(cos(q2)*cos(q3) - sin(q2)*sin(q3)) + a2*sin(q2) + a3*cos(q2)*sin(q3) + a3*cos(q3)*sin(q2)
```

evaluating then the position of point O_6 at the zero configuration

```
P=subs(p,{q1,q2,q3,q4,q5,q6},{0,0,0,0,0,0})


P =


            a1 + a2 + a3
              d5*sin(alf)
    d1 - d4 - d6 - d5*cos(alf)
```

similarly, at the zero configuration, the position of point H is

```
H =

    a1 + a2 + a3
              0
        d1 - d4
```