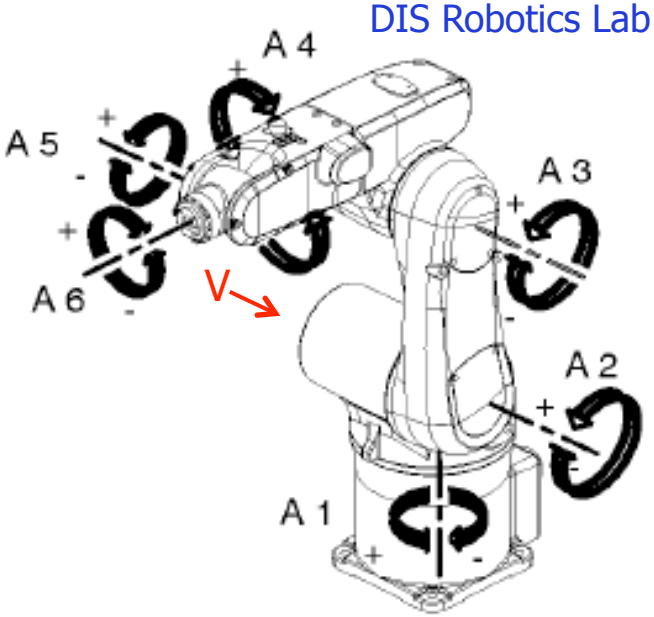
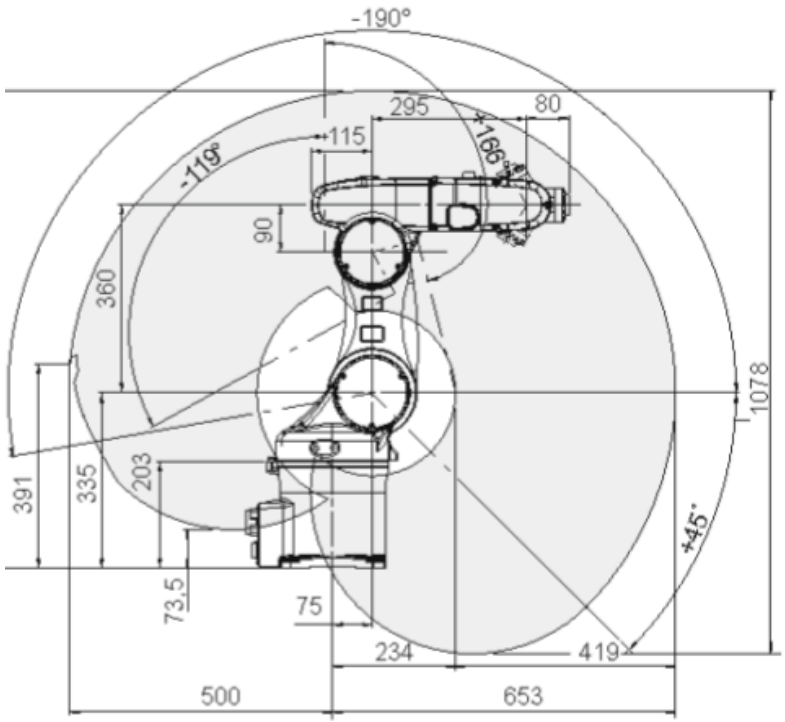
Exercise: KUKA KR5 Sixx R650

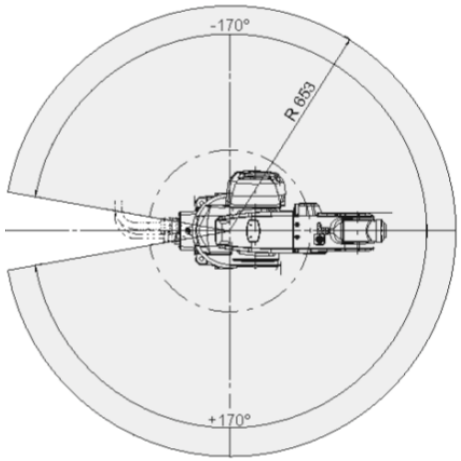
- 6R (offsets at shoulder and elbow, spherical wrist)

available at
DIS Robotics Lab





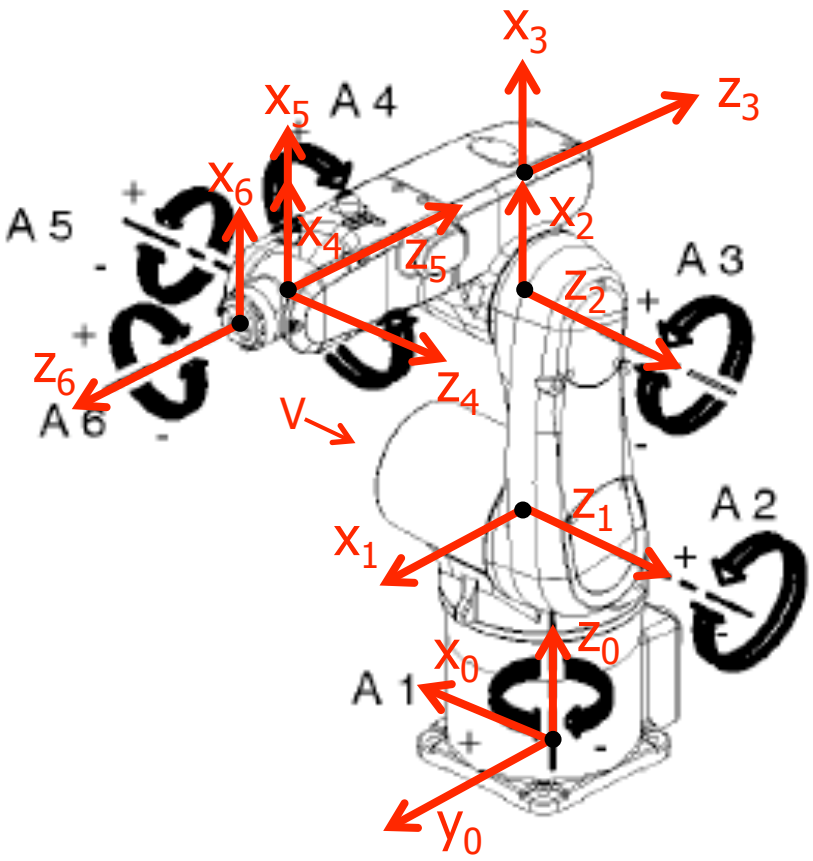
side view (from an observer in V)

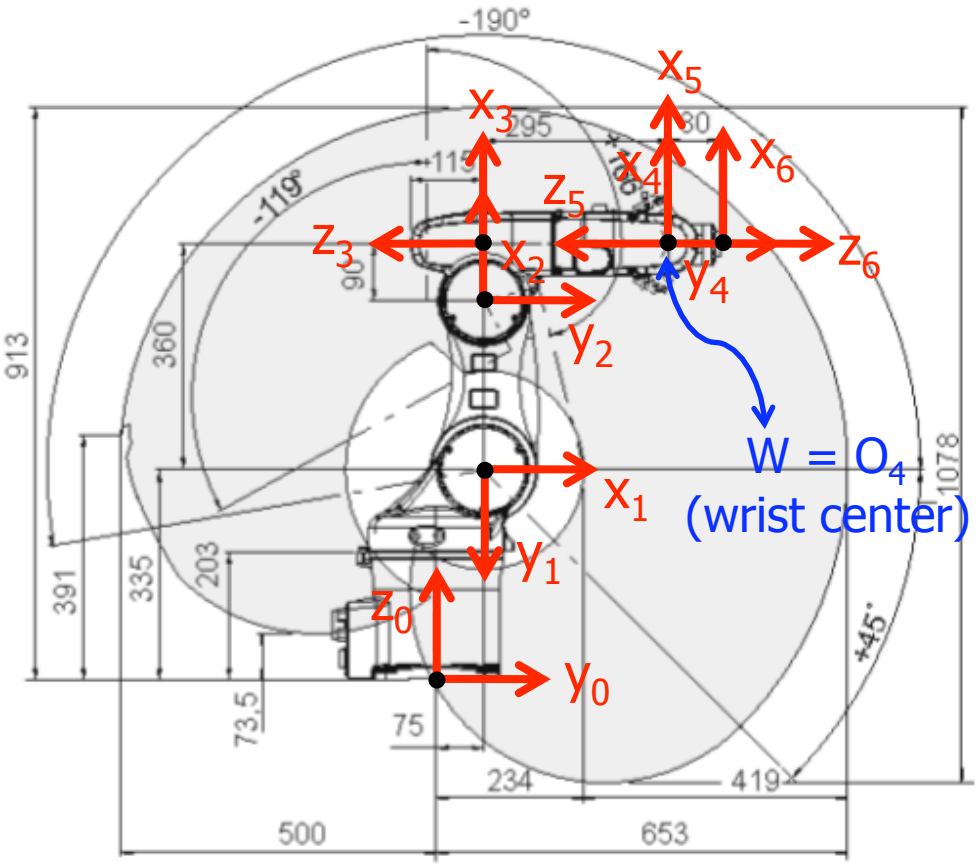


top view

- determine
  - frames and table of D-H parameters (be consistent with positive rotations indicated above by KUKA)
  - homogeneous transformation matrices
  - direct kinematics

Assignment of D-H frames





side view (from an observer in V)

Table of D-H parameters

i	α_i	d_i	a_i	θ_i
1	$-\pi/2$	d_1	a_1	$q_1=\pi/2$
2	0	0	a_2	$q_2=-\pi/2$
3	$\pi/2$	0	a_3	$q_3=0$
4	$-\pi/2$	d_4	0	$q_4=0$
5	$\pi/2$	0	0	$q_5=0$
6	π	d_6	0	$q_6=0$

with [in mm]

$d_1 = 335$ $\quad a_1 = 75$
$d_4 = -295$ $\quad a_2 = 270$
$d_6 = -80$ $\quad a_3 = 90$

D-H homogeneous matrices

output from Matlab (symbolic) program

```
A1 =

[ cos(q1),  0, -sin(q1), a1*cos(q1)]
[ sin(q1),  0,  cos(q1), a1*sin(q1)]
[       0, -1,        0,         d1]
[       0,  0,        0,          1]


A2 =

[ cos(q2), -sin(q2), 0, a2*cos(q2)]
[ sin(q2),  cos(q2), 0, a2*sin(q2)]
[       0,        0, 1,         0]
[       0,        0, 0,         1]


A3 =

[ cos(q3), 0,  sin(q3), a3*cos(q3)]
[ sin(q3), 0, -cos(q3), a3*sin(q3)]
[       0, 1,        0,         0]
[       0, 0,        0,         1]
```

```
A4 =

[ cos(q4),  0, -sin(q4),  0]
[ sin(q4),  0,  cos(q4),  0]
[       0, -1,        0, d4]
[       0,  0,        0,  1]


A5 =

[ cos(q5), 0,  sin(q5), 0]
[ sin(q5), 0, -cos(q5), 0]
[       0, 1,        0, 0]
[       0, 0,        0, 1]


A6 =

[ cos(q6),  sin(q6),  0,  0]
[ sin(q6), -cos(q6),  0,  0]
[       0,        0, -1, d6]
[       0,        0,  0,  1]
```

Direct kinematics - 1

output from Matlab (symbolic) program

$$T = A_1(q_1)A_2(q_2)A_3(q_3)A_4(q_4)A_5(q_5)A_6(q_6) = \left[\begin{array}{ccc|c} n & s & a & p \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

```
n =

 - sin(q1)*(cos(q4)*sin(q6) + cos(q5)*cos(q6)*sin(q4)) - cos(q2 + q3)*cos(q1)*(sin(q4)*sin(q6) - cos(q4)*cos(q5)*cos(q6)) - sin(q2 + q3)*cos(q1)*cos(q6)*sin(q5)
   cos(q1)*(cos(q4)*sin(q6) + cos(q5)*cos(q6)*sin(q4)) - cos(q2 + q3)*sin(q1)*(sin(q4)*sin(q6) - cos(q4)*cos(q5)*cos(q6)) - sin(q2 + q3)*cos(q6)*sin(q1)*sin(q5)
                                                         sin(q2 + q3)*(sin(q4)*sin(q6) - cos(q4)*cos(q5)*cos(q6)) - cos(q2 + q3)*cos(q6)*sin(q5)

s =

 sin(q1)*(cos(q4)*cos(q6) - cos(q5)*sin(q4)*sin(q6)) + cos(q2 + q3)*cos(q1)*(cos(q6)*sin(q4) + cos(q4)*cos(q5)*sin(q6)) - sin(q2 + q3)*cos(q1)*sin(q5)*sin(q6)
 cos(q2 + q3)*sin(q1)*(cos(q6)*sin(q4) + cos(q4)*cos(q5)*sin(q6)) - cos(q1)*(cos(q4)*cos(q6) - cos(q5)*sin(q4)*sin(q6)) - sin(q2 + q3)*sin(q1)*sin(q5)*sin(q6)
                                                                   - sin(q2 + q3)*(cos(q6)*sin(q4) + cos(q4)*cos(q5)*sin(q6)) - cos(q2 + q3)*sin(q5)*sin(q6)

                     a =

                        sin(q1)*sin(q4)*sin(q5) - sin(q2 + q3)*cos(q1)*cos(q5) - cos(q2 + q3)*cos(q1)*cos(q4)*sin(q5)
                      - sin(q2 + q3)*cos(q5)*sin(q1) - cos(q1)*sin(q4)*sin(q5) - cos(q2 + q3)*cos(q4)*sin(q1)*sin(q5)
                                                               sin(q2 + q3)*cos(q4)*sin(q5) - cos(q2 + q3)*cos(q5)

p =

 cos(q1)*(a1 + a3*cos(q2 + q3) + a2*cos(q2)) + sin(q2 + q3)*cos(q1)*(d4 + d6*cos(q5)) - d6*sin(q1)*sin(q4)*sin(q5) + d6*cos(q2 + q3)*cos(q1)*cos(q4)*sin(q5)
 sin(q1)*(a1 + a3*cos(q2 + q3) + a2*cos(q2)) + sin(q2 + q3)*sin(q1)*(d4 + d6*cos(q5)) + d6*cos(q1)*sin(q4)*sin(q5) + d6*cos(q2 + q3)*cos(q4)*sin(q1)*sin(q5)
                                               d1 + cos(q2 + q3)*(d4 + d6*cos(q5)) - a3*sin(q2 + q3) - a2*sin(q2) - d6*sin(q2 + q3)*cos(q4)*sin(q5)
```

$$W_{hom} = A_1(q_1)A_2(q_2)A_3(q_3)A_4(q_4) \cdot [0\ 0\ 0\ 1]^T = A_1(q_1)A_2(q_2)A_3(q_3) \cdot [0\ 0\ d_4\ 1]^T$$

or $W = p(q) - d_6 a(q)$

```
W =

 cos(q1)*(a1 + a3*cos(q2 + q3) + d4*sin(q2 + q3) + a2*cos(q2))
 sin(q1)*(a1 + a3*cos(q2 + q3) + d4*sin(q2 + q3) + a2*cos(q2))
           d1 + d4*cos(q2 + q3) - a3*sin(q2 + q3) - a2*sin(q2)
```

Direct kinematics - 2

numerical evaluation (Matlab) in the shown robot configuration ...

```
>> q1=pi/2; q2=-pi/2; q3=0; q4=0; q5=0; q6=0;
>> subs(T)

ans =

[ 0, 1, 0,              0]
[ 0, 0, 1, a1 - d4 - d6]
[ 1, 0, 0, a2 + a3 + d1]
[ 0, 0, 0,              1]

>> subs(W)

ans =

            0
      a1 - d4
 a2 + a3 + d1
```

... and with the numerical values of the other D-H parameters ...

```
>> a1=75;a2=270;a3=90;d1=335;d4=-295;d6=-80;
>> subs(T)

ans =

    0.0000    1.0000    0.0000    0.0000
    0.0000   -0.0000    1.0000  450.0000
    1.0000         0   -0.0000  695.0000
         0         0         0    1.0000

>> subs(W)

ans =

    0.0000
  370.0000
  695.0000
```