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## **Flexible joint robots: Model-based control revisited**

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# Summary

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- **A world of soft robots**
  - flexible joints, serial elastic actuation (SEA), variable stiffness actuation (VSA), distributed link flexibility
  - lightweight robots with flexible joints in physical Human-Robot Interaction (pHRI)
- **Dynamic modeling of flexible joint manipulators**
  - ... with few comments on its properties
- **Classical control tasks and their solution**
  - full-state feedback linearization design for **trajectory tracking**
  - **regulation** with partial state feedback and gravity compensation
- **Model-based design based on feedback equivalence**
  - exact gravity cancellation
  - damping injection on the link side
  - environment interaction via generalized impedance control
- **Outlook**

# Classes of soft robots

## Robots with **elastic joints**

- design of **lightweight** robots with **stiff links** for end-effector accuracy
- **compliant elements** absorb impact energy
  - soft coverage of links (safe bags)
  - elastic transmissions (HD, cable-driven, ...)
- **elastic joints decouple instantaneously** the *larger* inertia of the driving motors from *smaller* inertia of the links (involved in contacts/collisions!)
- *relatively* soft joints need more **sensing** (e.g., joint torque) and better **control** to compensate for static deflections and dynamic vibrations

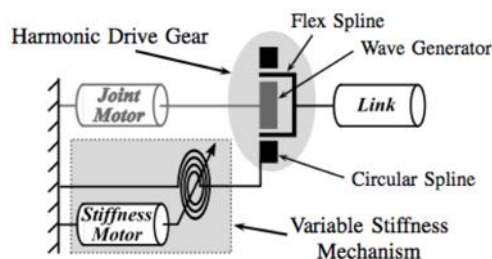


➔ **torque-controlled** robots (DLR LWR-III, KUKA LWR-IV & iiwa, Franka, ...)

# Classes of soft robots

## Robots with **Variable Stiffness Actuation (VSA)**

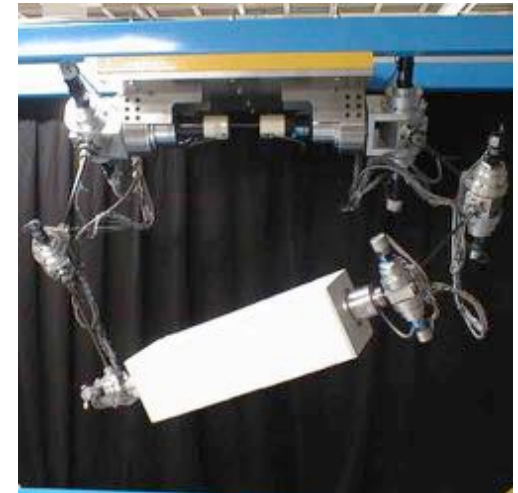
- uncertain **interaction** with dynamic environments (say, *humans*) requires to adjust online the compliant behavior and/or to control contact forces
  - **passive** joint elasticity & **active** impedance control used **in parallel**
- nonlinear flexible joints with **variable (controlled) stiffness** work at best
  - can be made *stiff when moving slow (performance)*, *soft when fast (safety)*
  - enlarge the set of achievable robot compliance in a task-oriented way
  - feature also **robustness**, optimal **energy use**, **explosive motion** tasks, ...



# Classes of soft robots

## Robots with **flexible links**

- **distributed** link deformations
  - design of **very long** and **slender** arms needed in the application
  - use of **lightweight** materials to save weight/costs
  - due to large payloads (viz. large contact forces) and/or high motion speed
- as for joint elasticity, neglecting link flexibility will limit **static** (steady-state error) or **dynamic** (vibrations, poor tracking) performance
- extra control issue due to **non-minimum phase** nature of the outputs of interest w.r.t. the command inputs ... “move in the opposite direction!”





# A matter of terminology ...

Different sources of elasticity, though similar robotic systems

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- **elastic joints vs. SEA (Serial Elastic Actuators)**
  - based on the same physical phenomenon: **compliance in actuation**
  - compliance added **on purpose** in SEA, mostly a **disturbance** in elastic joints
  - different **range** of stiffness: **5-10K** Nm/rad down to **0.2-1K** Nm/rad in SEA
- **joint deformation is often considered in the linear domain**
  - modeled as a **concentrated** torsional spring with constant stiffness at the joint
  - nonlinear flexible joints share similar control properties
  - **nonlinear** stiffness characteristics are needed instead in VSA
  - a (serial or antagonistic) VSA working at constant stiffness **is** an elastic joint
- **flexible joint robots are classified as underactuated mechanical systems**
  - have **less commands** than generalized coordinates
  - **non-collocation** of command inputs and dynamic effects to be controlled
  - however, they are **controllable** in the first approximation (the *easy case!*)

# Exploiting joint elasticity in pHRI

Detection and selective reaction in torque control mode, based on **residuals**

- **collision detection & reaction** for safety (model-based + joint torque sensing)

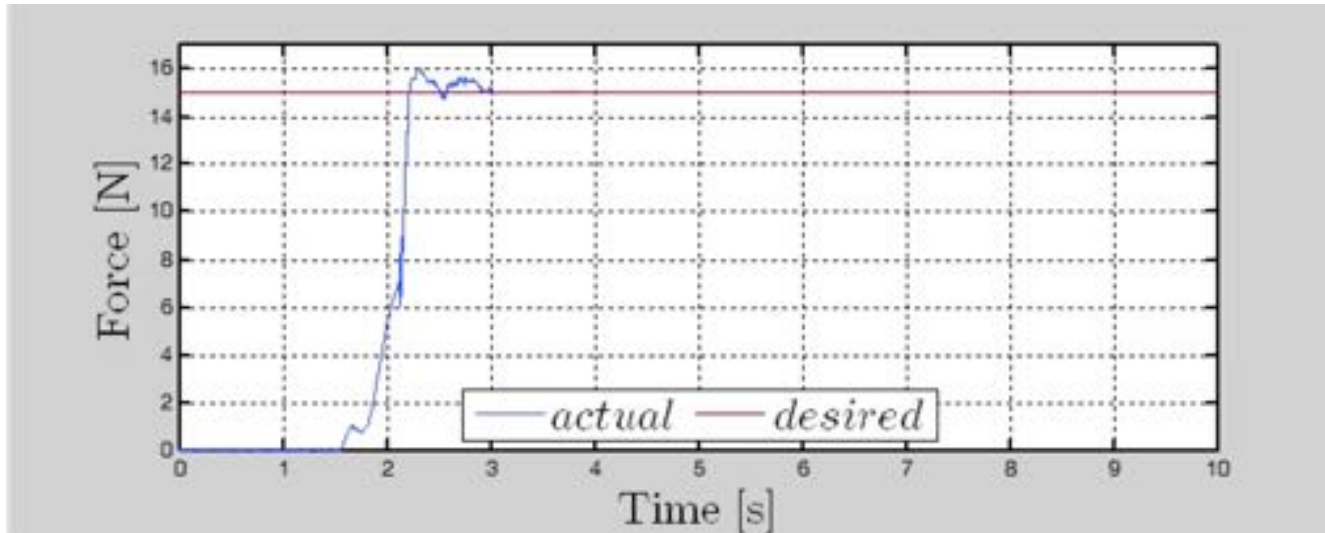
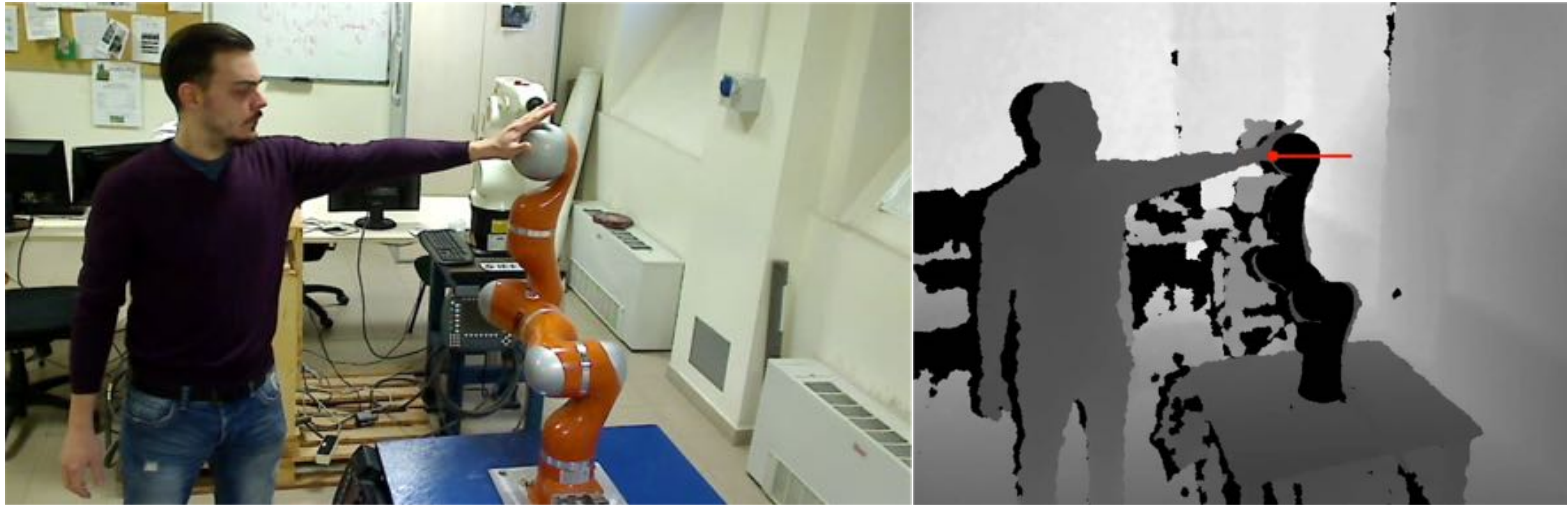


[De Luca  
*et al*, 2006;  
Haddadin  
*et al*, 2017]

# Exploiting joint elasticity in pHRI

## Human-robot collaboration in torque control mode

- contact force estimation & control (virtual force sensor, anywhere/anytime)



[Magrini  
*et al*, 2015]



# Dynamic modeling

## Lagrangian formulation (so-called **reduced** model of Spong)

- open chain robot with N elastic joints and N rigid links, driven by electrical actuators
- use N **motor variables**  $\theta$  (as reflected through the gear ratios) and N **link variables**  $q$

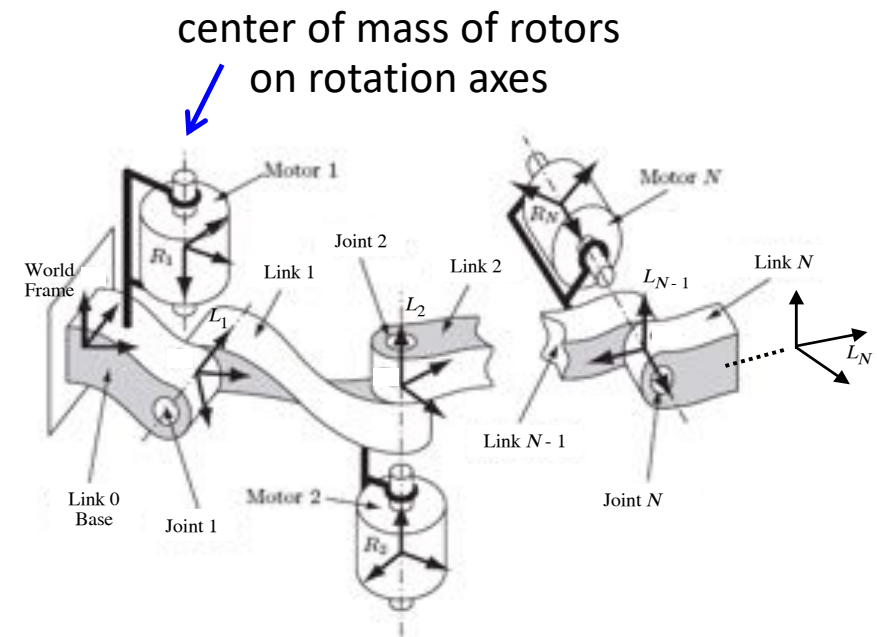
### ■ assumptions

A1) small displacements at joints

A2) axis-balanced motors

A3) each motor is mounted on the robot  
in a position **preceding** the driven link

A4) **no inertial couplings** between motors and links



A4)  $\Rightarrow$   $2N \times 2N$   
inertia matrix  
Is block diagonal

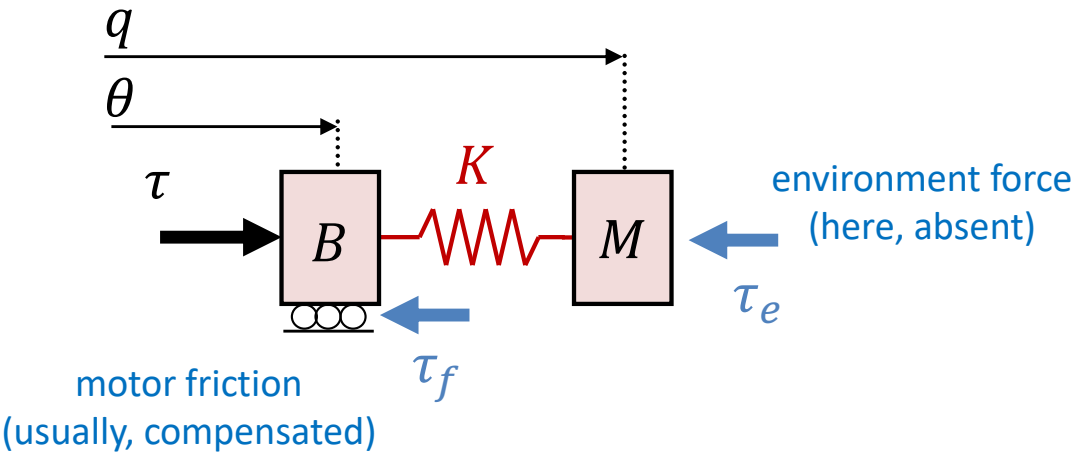
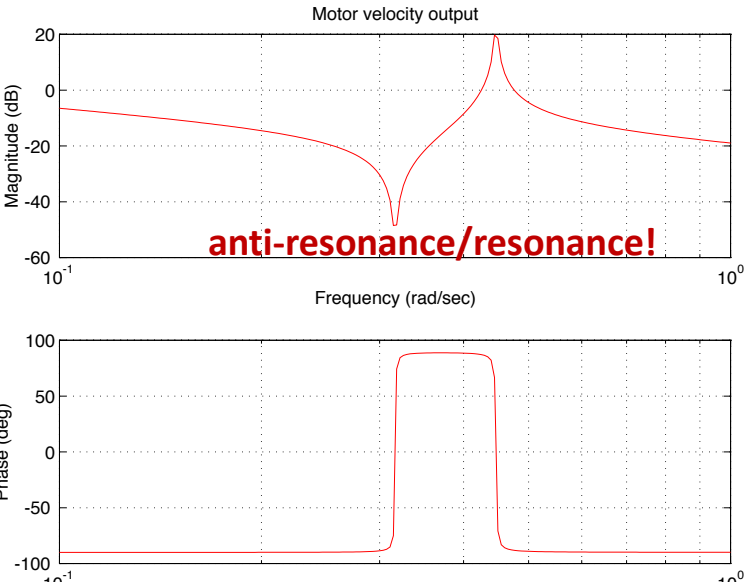
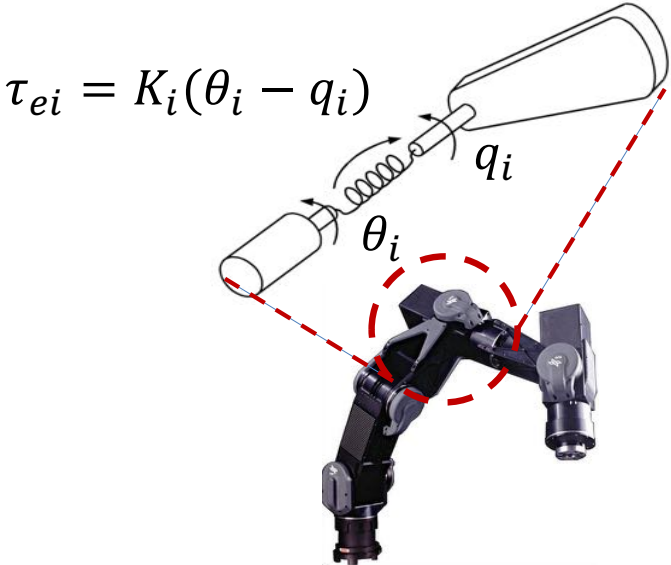
A2)  $\Rightarrow$  inertia matrix  
and gravity vector are  
independent from  $\theta$

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

**link equation**  
**motor equation**

# Single elastic joint

## Transfer functions of interest



$$P_{\text{motor}}(s) = \frac{\theta(s)}{\tau(s)} = \frac{Ms^2 + K}{MBs^2 + (M + B)K} \frac{1}{s^2}$$

- system with zeros and relative degree = 2
- **passive** (zeros always precede poles on the imaginary axis)
- stabilization can be achieved via output  $\theta$  feedback

$$P_{\text{link}}(s) = \frac{q(s)}{\tau(s)} = \frac{K}{MBs^2 + (M + B)K} \frac{1}{s^2}$$

- **NO zeros!!**
- maximum relative degree = 4



# Feedback linearization

For accurate trajectory tracking tasks

- the link position  $q$  is a **linearizing (flat) output**

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} \iff \boxed{q^{(4)} = u}$$

- differentiating twice the link equation and using the motor acceleration yields

$$\tau = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1} \left( 2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2} (C\dot{q} + g(q)) \right)$$

- an **exactly** linear and I/O decoupled closed-loop system is obtained
  - to be stabilized with standard techniques for linear dynamics (pole placement, LQ, ...)
- requires **higher derivatives** of  $q$   $\text{-----}$   $\boxed{q, \dot{q}, \ddot{q}, q^{(3)}}$
- however, these can be computed **from the model** using the state measurements
- requires **higher derivatives** of the dynamics components  $\text{-----}$   $\boxed{\ddot{M}, \ddot{C}, \ddot{g}}$
- A  $O(N^3)$  **Newton-Euler** recursive numerical algorithm is available for this problem

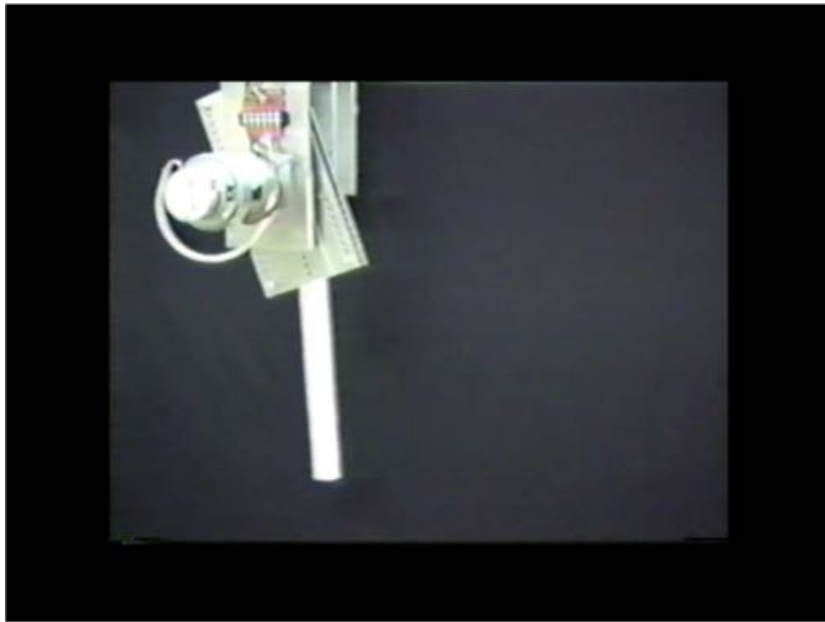
# Feedback linearization

Based on the **rigid model** only vs. when including **joint elasticity**

$$\tau = M(q)(\ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P(q_d - q)) + C(q, \dot{q})\dot{q} + g(q)$$

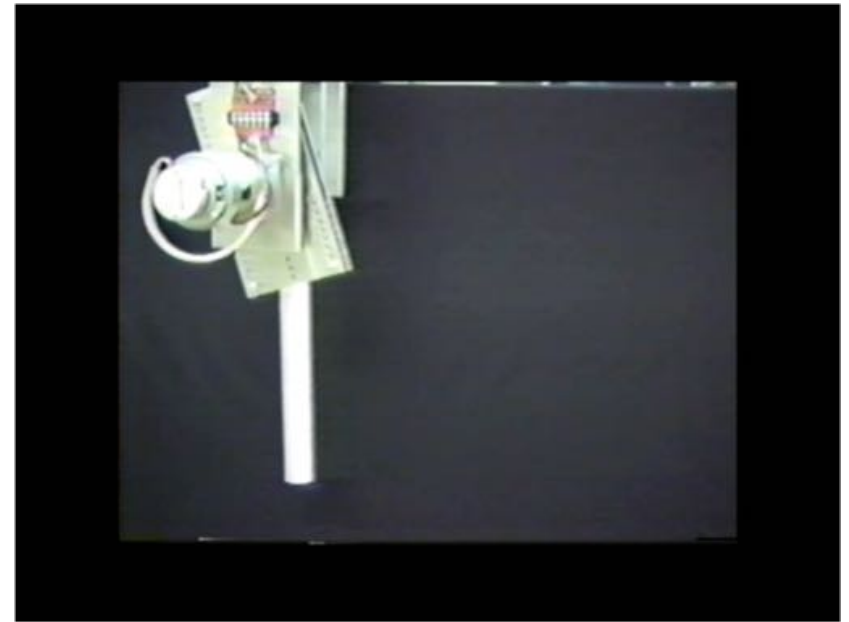
$$\tau = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1} \left( 2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q)) \right)$$

$$u = \left( q_d^{[4]} + K_J(\ddot{q}_d - \ddot{q}) + K_A(\ddot{q}_d - \ddot{q}) + K_D(\dot{q}_d - \dot{q}) + K_P(q_d - q) \right)$$



**rigid** computed torque

[Spong, 1986]



**elastic joint** feedback linearization

# Feedback linearization

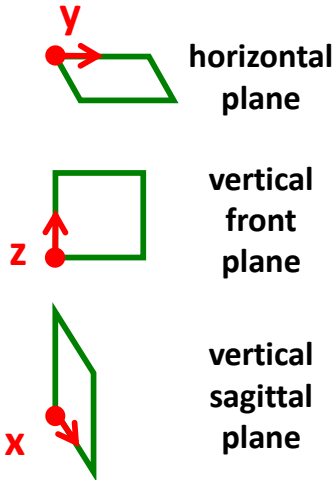
Benefits on an industrial KUKA KR-15/2 robot (235 kg) with **joint elasticity**



conventional industrial robot control



three squares in:



[Thümmel, 2007]



feedback linearization + high-damping



trajectory tracking with model-based control



# Regulation tasks

Using a minimal **PD+** action on the motor side

for a desired **constant** link position  $q_d$

- evaluate the associated desired motor position  $\theta_d$  at steady state
- collocated (**partial state**) feedback preserves passivity, with **stiff  $K_P$  gain dominating gravity**
- focus on the term for **gravity compensation** (acting on link side) from motor measurements

$$\theta_d = q_d + K^{-1}g(q_d) \quad \tau = \tau_g + K_P(\theta_d - \theta) - K_D\dot{\theta} \quad K_D > 0$$

$\tau_g$	gain criteria for stability	
$g(q_d)$	$\lambda_{\min} \begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix} > \alpha$	[Tomei, 1991]
$g(\theta - K^{-1}g(q_d))$	$\lambda_{\min} \begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix} > \alpha$	[De Luca, Siciliano, Zollo, 2004]
$g(\bar{q}(\theta)), \bar{q}(\theta): g(\bar{q}) = K(\theta - \bar{q})$	$K_P > 0, \lambda_{\min}(K) > \alpha$	[Ott, Albu-Schäffer, 2004]
$g(q) + BK^{-1}\ddot{g}(q)$	$K_P > 0, K > 0$	[De Luca, Flacco, 2010]

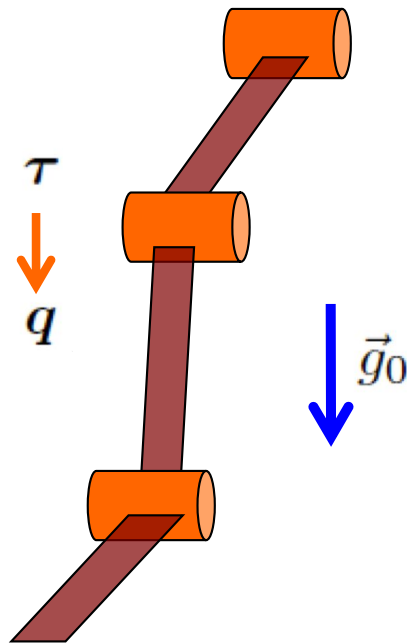
exact gravity cancellation  
(with full state feedback)

$$\alpha = \max\left(\left\|\frac{\partial g(q)}{\partial q}\right\|\right)$$

# Exact gravity cancellation

## A slightly different view

- for rigid robots this is **trivial**, due to **collocation**



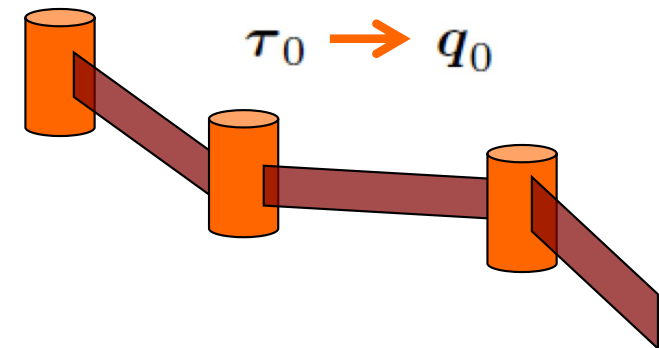
$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = \tau$$

$$\tau = \tau_g + \tau_0$$

➔

$$\tau_g = g(q)$$

$$q \equiv q_0$$

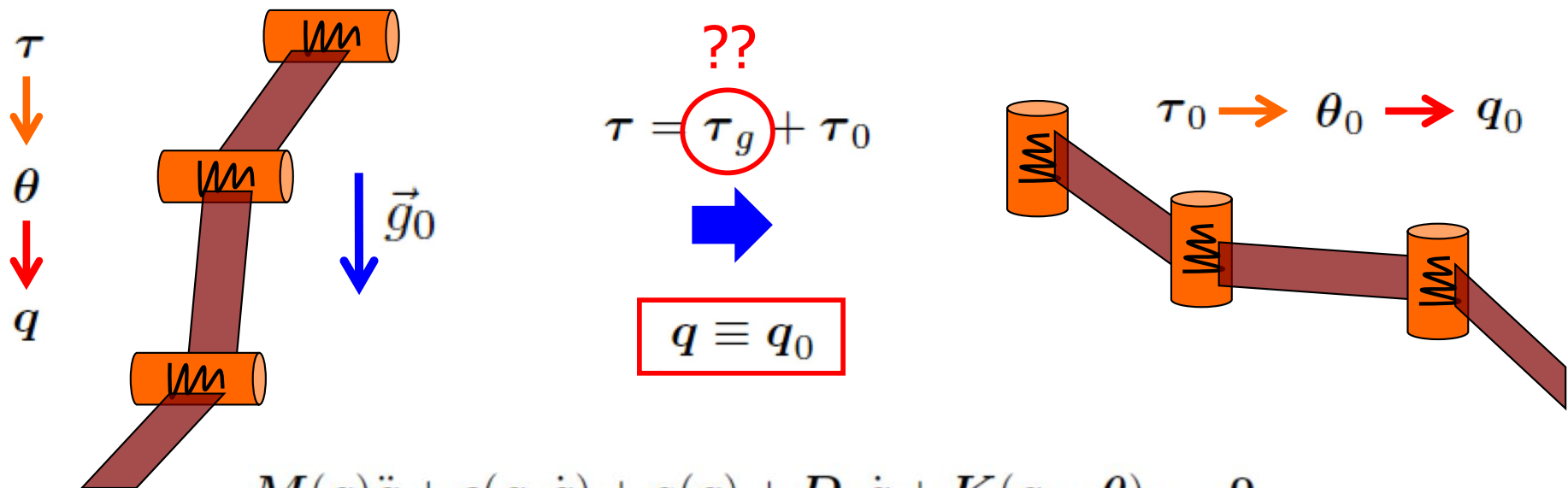


$$M(q)\ddot{q} + c(q, \dot{q}) = \tau_0$$

# Exact gravity cancellation

... based on the concept of **feedback equivalence** between nonlinear systems

- for elastic joint robots, **non-collocation** of input torque and gravity term



$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) + D_q\dot{q} + K(q - \theta) = 0$$

$$B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) = \tau$$

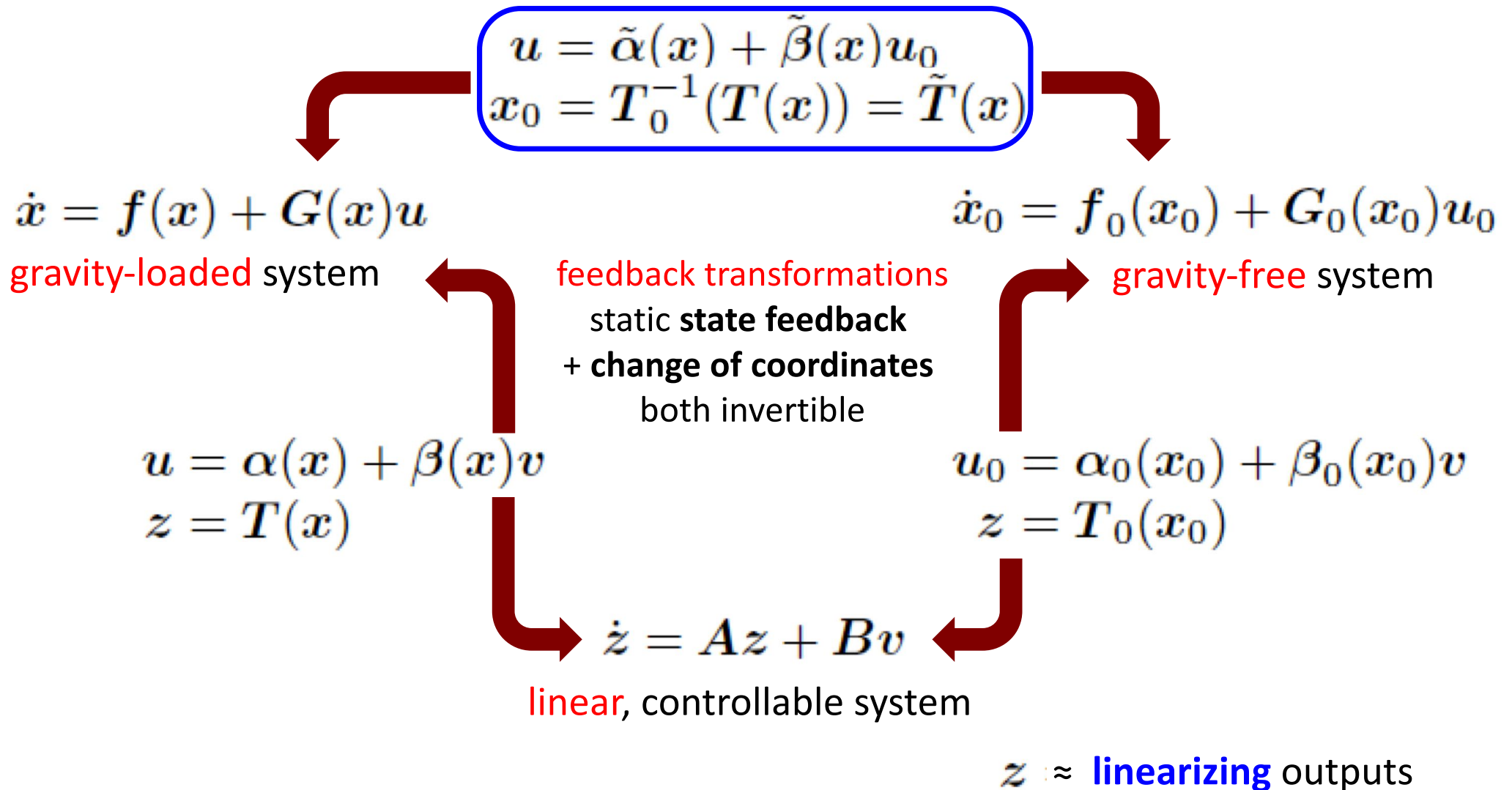
$$\tau_g = g(q) + D_\theta K^{-1} \dot{g}(q) + BK^{-1} \ddot{g}(q)$$





# Feedback equivalence

Exploit the system property of being feedback linearizable (**without** forcing it!)

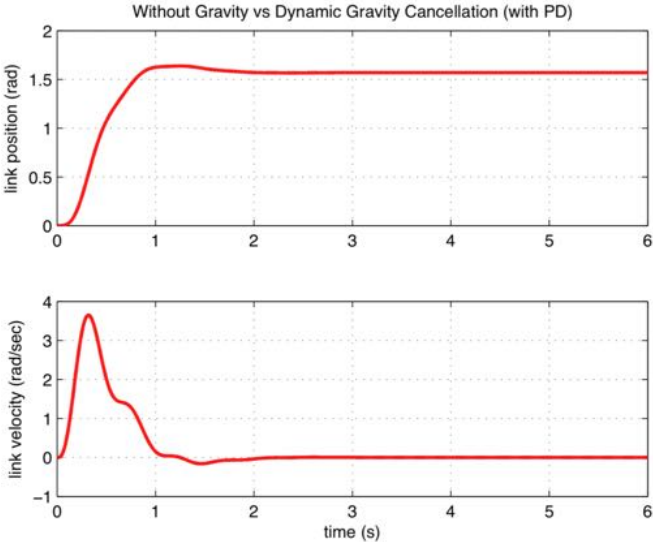




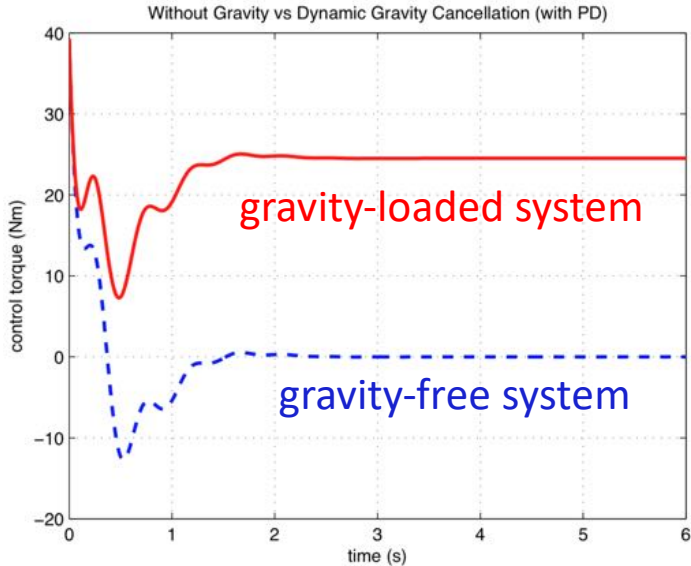
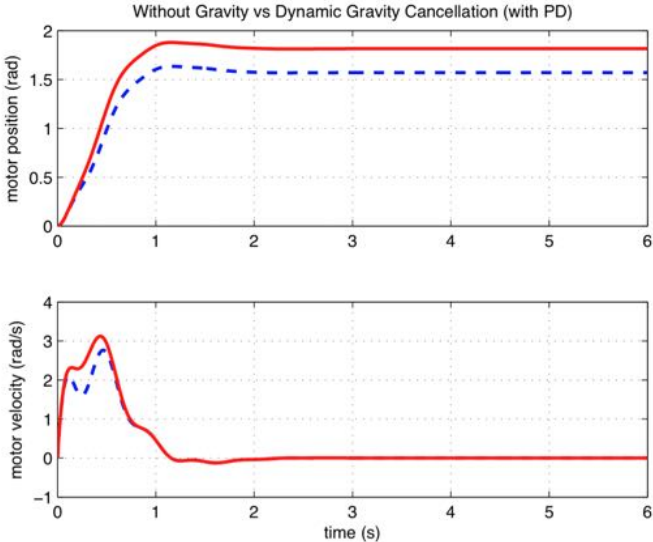
# A global PD-type regulator

Exact gravity cancellation + PD law on **modified** motor variables: A **1-DOF** arm

identical link behavior



different motor behavior



total control torque

gravity-loaded system under PD + gravity cancellation vs. gravity-free system under PD (with **same gains**)

$K_P > 0$     $K > 0$

works **without** strictly positive lower bounds (**good** also for **VSA!**)

# Vibration damping on lightweight robots

DLR-III or KUKA LWR-IV with relatively **low** joint elasticity (use of Harmonic Drives)



Vibration damping **OFF**



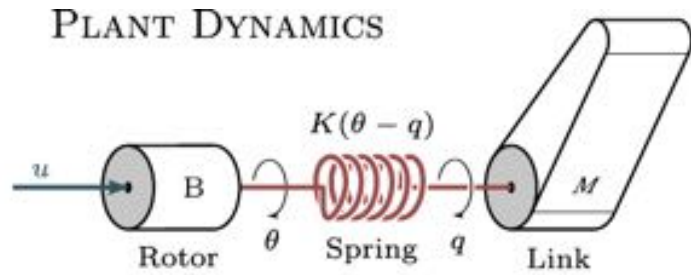
Cartesian vibration damping **ON**

[Albu Schäffer *et al*, 2007]

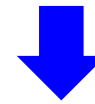
For relatively **large** joint elasticity (low stiffness), as encountered in VSA systems, vibration damping via joint torque feedback + motor damping is **insufficient** for high performance!

# Damping injection on the link side

Method for the **VSA-driven** bimanual humanoid torso **David**



$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

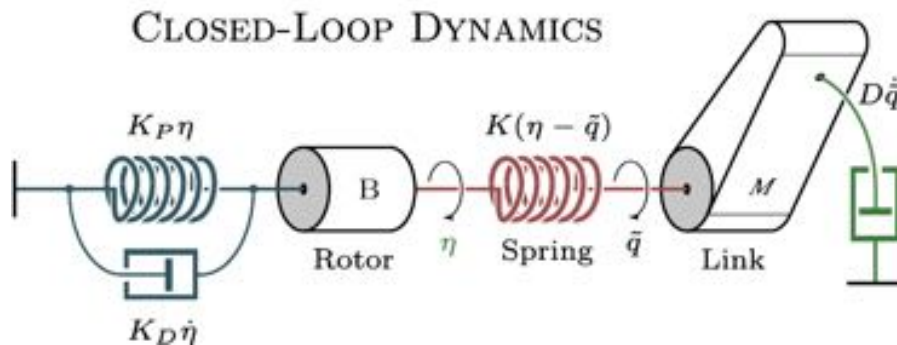


$$K(q - \theta) = K(q - \theta_0) + D\dot{q}$$

state transformation

$$\tau = \tau_0 - D\dot{q} - BK^{-1}D\ddot{q}$$

feedback control



$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta}_0 \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta_0) \\ K(\theta_0 - q) \end{pmatrix} = \begin{pmatrix} -D\dot{q} \\ \tau_0 \end{pmatrix}$$

- same principle of **feedback equivalence** (including state transformation)
- **ESP** = Elastic Structure Preserving control by DLR [\[Keppler et al, 2016\]](#)
- generalizations to **trajectory tracking**, to nonlinear joint flexibility, and to visco-elastic joints

# Damping injection on the link side

Method for **VSA-driven** bimanual humanoid torso **David** at DLR

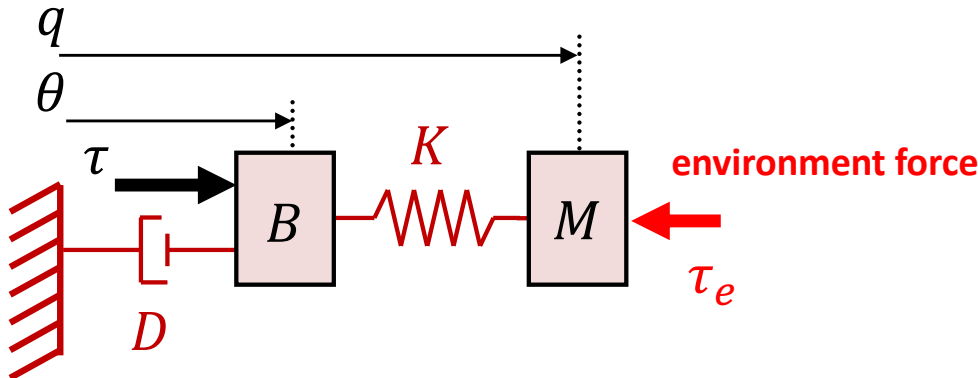


[Keppler *et al*, 2017]



# Environment interaction via impedance control

**Matching** a generalized (fourth order) impedance model: A simple **1-DOF** case



$$M\ddot{q} + K(q - \theta) = \tau_e$$

$$B\ddot{\theta} + D\dot{\theta} + K(\theta - q) = \tau$$



feedback control

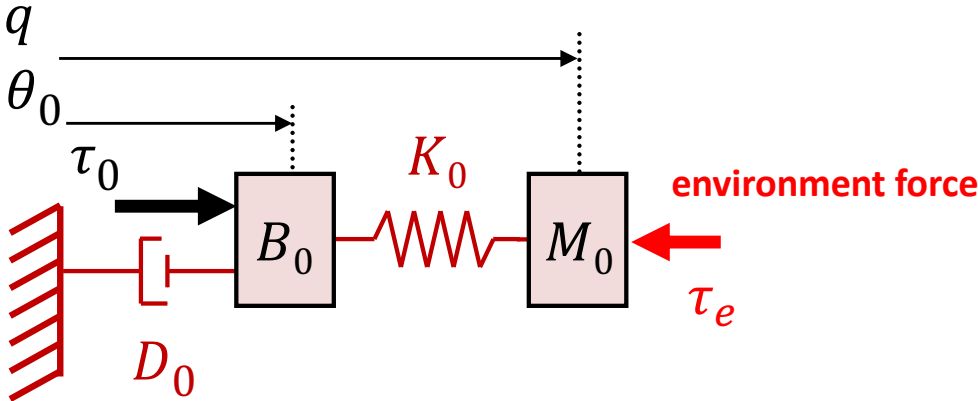
assume that  $M_0 = M$   
in order to avoid **derivatives**  
of the measured force  $\tau_e$

$$\tau = K(\theta - q) + D\dot{\theta} - BK^{-1} \left\{ \begin{array}{l} (K - K_0)M^{-1}(\tau_e + K(\theta - q)) \\ + K_0B_0^{-1}(\tau_0 - D_0\dot{\theta}_0 - K(\theta - q)) \end{array} \right\}$$

$$\dot{\theta}_0 = \dot{q} + KK_0^{-1}(\dot{\theta} - \dot{q})$$



state transformation



$$M_0\ddot{q} + K_0(q - \theta_0) = \tau_e$$

$$B_0\ddot{\theta}_0 + D_0\dot{\theta}_0 + K_0(\theta_0 - q) = \tau_0$$

- again, by the principle of **feedback equivalence** (including the state transformation)



# Outlook

## Control of flexible robots in 2020+

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- **Mature field revamped by a new “explosion” of interest**
    - simpler control laws for compliant and soft robots are very welcome
    - sensing requirements could be a bottleneck
    - combine (learned) feedforward and feedback to achieve robustness
    - iterative learning on repetitive tasks is available for flexible manipulators
    - optimal control (min time, min energy, max force, ...) still open for fun
  - **Revisiting model-based control design**
    - do not fight too much against the natural dynamics of the system
      - it is unwise to stiffen what was designed/intended to be soft on purpose
    - still, don't give up too much of desirable performance!
  - **Ideas assessed for joint elasticity may migrate to many application domains and other classes of soft-bodied robots**
    - locomotion, shared manipulation, physical interaction in complex tasks, ...
    - keep in mind intrinsic constraints and control limitations (e.g., instabilities in the system inversion of tip trajectories for flexible link robots)
-