

***IROS 2018 Workshop on Soft Robotic Modeling and Control:  
Bringing Together Articulated Soft Robots and Soft-Bodied Robots***

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## **A review on the control of flexible joint manipulators**

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# Summary

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- **Motivations and definitions**
    - elastic/flexible joint, serial elastic actuation (SEA), variable stiffness actuation (VSA)
    - concentrated, collocated and distributed flexibility
  - **Dynamic modeling of elastic joint manipulators**
    - control properties
    - differences with flexibility in the links
  - **Regulation tasks**
    - partial state vs. full state feedback
    - PD+ control laws, with different gravity compensation/cancellation techniques
  - **Trajectory tracking tasks**
    - inverse dynamics (feedforward)
    - feedback linearization
    - torque control
  - **Latest approach**
    - least modification of elastic dynamics: exact gravity cancellation, link damping, ESP ...
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# Classes of soft robots

## Robots with **elastic joints**

- **lightweight** but **stiff link** design reduces robot inertia and preserves kinematic accuracy at end-effector level
- **compliant elements** can absorb impact energy
  - soft coverage of links (safe bags)
  - elastic transmissions/joints (HD, cable-driven, ...)
- **elastic joints decouple instantaneously** the *larger* inertia of the driving motors from the *smaller* inertia of the links (where collisions occur!)
  - robots with *relatively soft* joints need more *sensing* and better *control* laws to compensate for static deflections and dynamic vibrations

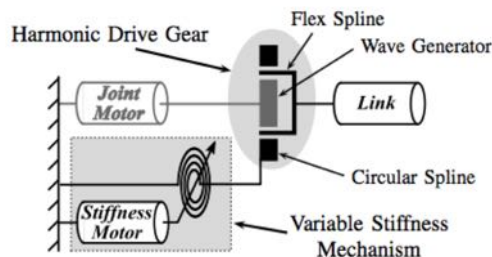
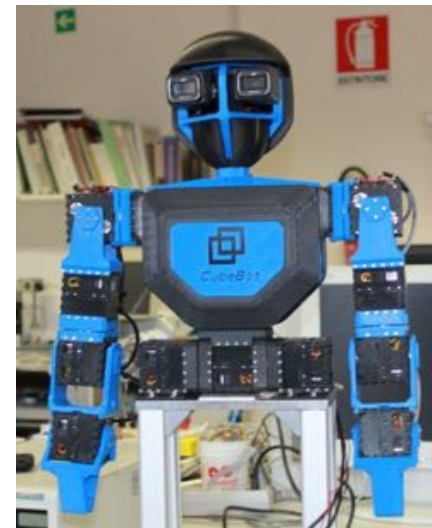


**→ torque-controlled robots (DLR LWR-III, KUKA LWR 4, KUKA iiwa, ...)**

# Classes of soft robots

## Robots with **Variable Stiffness Actuation (VSA)**

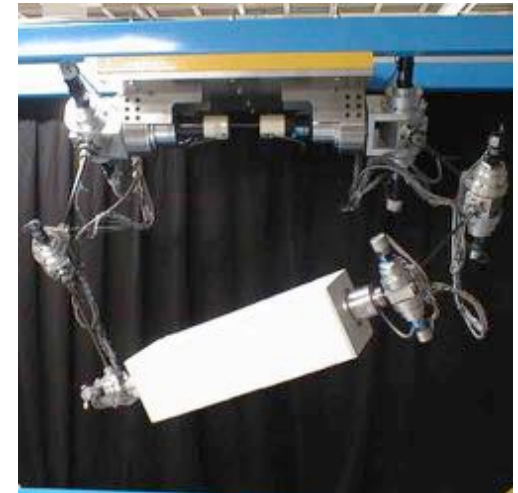
- uncertain/dynamic interaction with the environment requires to adjust the compliant behavior of the robot and/or to control contact forces
  - **passive** joint elasticity & **active** impedance control used **in parallel**
- **nonlinear** flexible joints with **variable (controlled) stiffness** do their best:
  - can be made *stiff when moving slow (performance)*, *soft when fast (safety)*
  - enlarge the set of achievable task-oriented compliance matrices
  - feature also: robustness, energy optimization, explosive motion tasks, ...



# Classes of soft robots

## Robots with **flexible links**

- **distributed** link deformations in robots
  - need to design **very long** and **slender** arms for the application
  - use of **lightweight** materials to save weight/costs
  - due to large payloads and/or high motion speed (or large contact forces)
- as for joint elasticity, neglecting link flexibility will limit **static** (steady-state error) or **dynamic** (vibrations, poor tracking) performance
- additional control problems due to the **non-collocation** of typical output quantities of interest w.r.t. the input commands





# Additional notes

...also terminology varies for the considered robots

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- **elastic joints vs. SEA (Serial Elastic Actuators)**
    - consider/use the same physical phenomenon: **compliance in actuation**
    - compliance added **on purpose** in SEA, mostly is **a disturbance** in elastic joints
    - different **range** of stiffness: **5-10K Nm/rad** down to **0.2-1K Nm/rad** in SEA
  - **joint torque sensors introduce joint elasticity!**
  - **joint deformation is often considered in the linear domain**
    - modeled as a **concentrated** torsional spring with constant stiffness at the joint
    - **nonlinear** flexible joints are handled too, and share similar control properties
    - **viscosity** may also be present (visco-elastic joints)
    - nonlinear stiffness characteristics are needed in VSA
  - **(serial or antagonistic) VSA working at constant stiffness are elastic joints**
  - **often classified as underactuated mechanical systems**
    - have less commands than generalized coordinates
    - however, are **controllable** in the first approximation (the easy case!)
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# Dynamic modeling

## Lagrangian formulation for the **complete** model

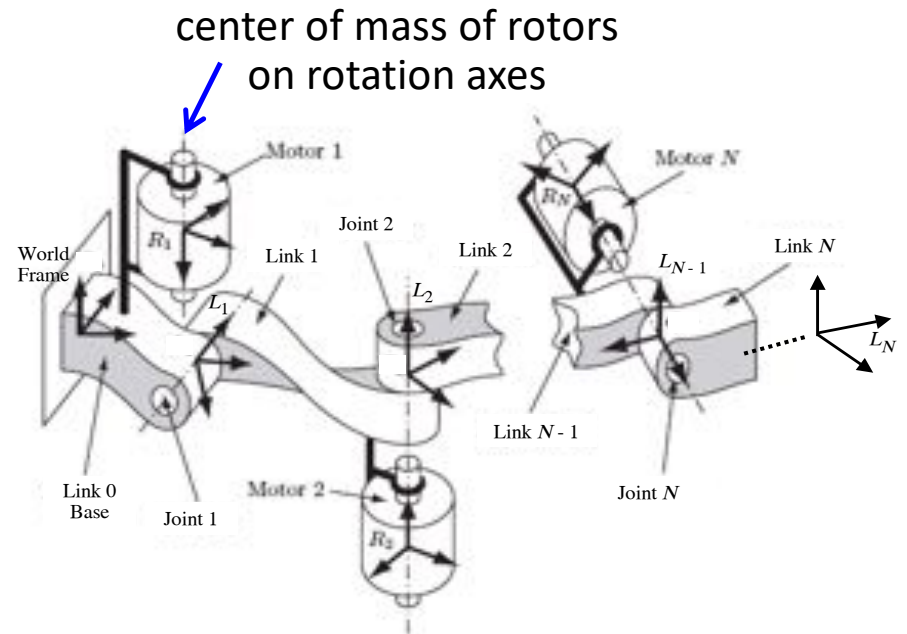
- open chain robot with  $N$  (rotary or prismatic) elastic joints and  $N$  rigid links, driven by electrical actuators
- use  $N$  **motor variables**  $\theta$  (as reflected through the gear ratios) and  $N$  **link variables**  $q$
- standing assumptions**

- A1) small displacements at joints
- A2) axis-balanced motors
- A3) each motor is mounted on the robot in a position **preceding** the driven link



$2N \times 2N$  (full) inertia matrix

A2)  $\Rightarrow$  inertia matrix and gravity vector independent from  $\theta$



$$\begin{bmatrix} M_L(q) & S(q) \\ S^T(q) & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} c_1(q, \dot{q}, \dot{\theta}) \\ c_2(q, \dot{q}) \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$

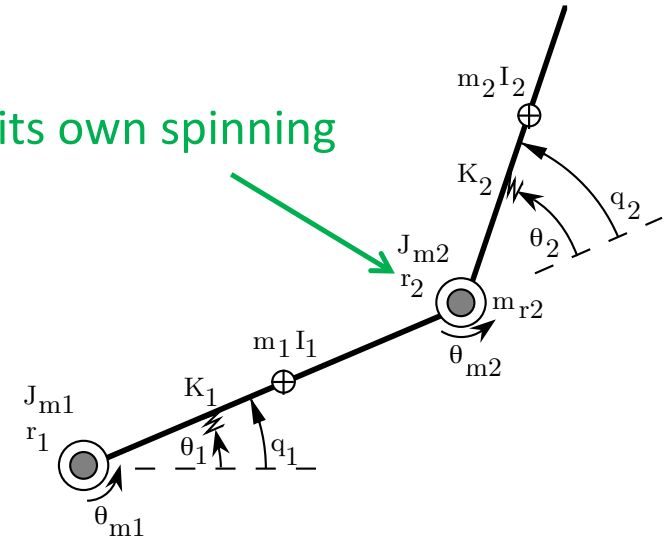
# Dynamic modeling

## Approximation for the **reduced** model (Spong 87)

- **simplifying** assumption

A4) the angular kinetic energy of each motor is due only to **its own spinning**

$$S(q) = 0$$



$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$

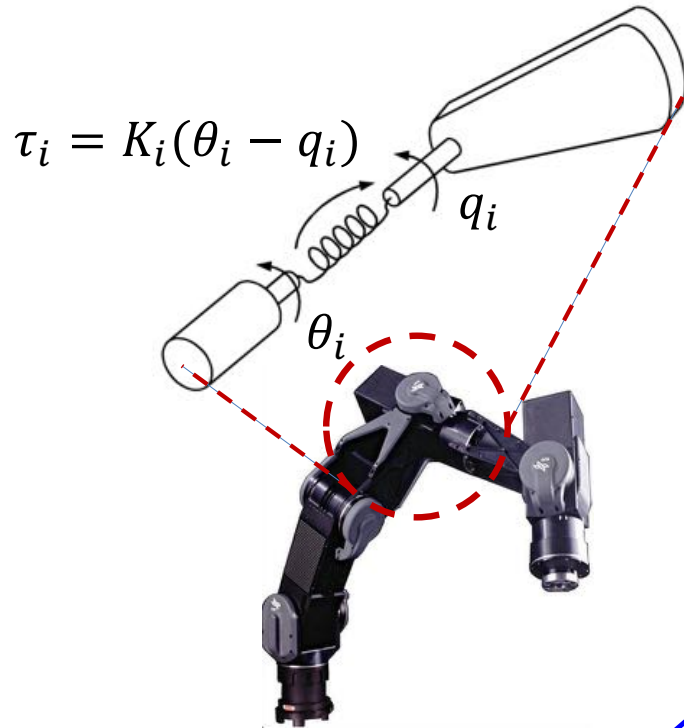
link equation  
motor equation

complete model	reduced model
inertial and stiffness couplings	only stiffness couplings
linearizable by <b>dynamic</b> state feedback [De Luca, Lucibello 98]	linearizable by <b>static</b> state feedback [Spong 87]
always valid (under assumptions A1-A3)	A4 valid when gear ratios are very high

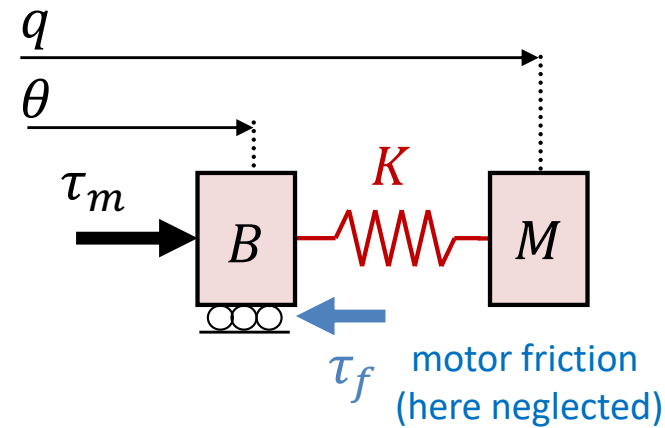


# Single elastic joint

## Transfer functions of interest



- with **viscous friction** on motor and/or link, complex pole/zero pairs are moved to the lhs of the  $C$ -plane



$$P_{\text{motor}}(s) = \frac{\theta(s)}{\tau_m(s)} = \frac{Ms^2 + K}{MBs^2 + (M + B)K} \frac{1}{s^2}$$

- controllable system with zeros
- passive (zeros always precede poles on the imaginary axis)
- stabilization can be achieved via output  $\theta$  feedback

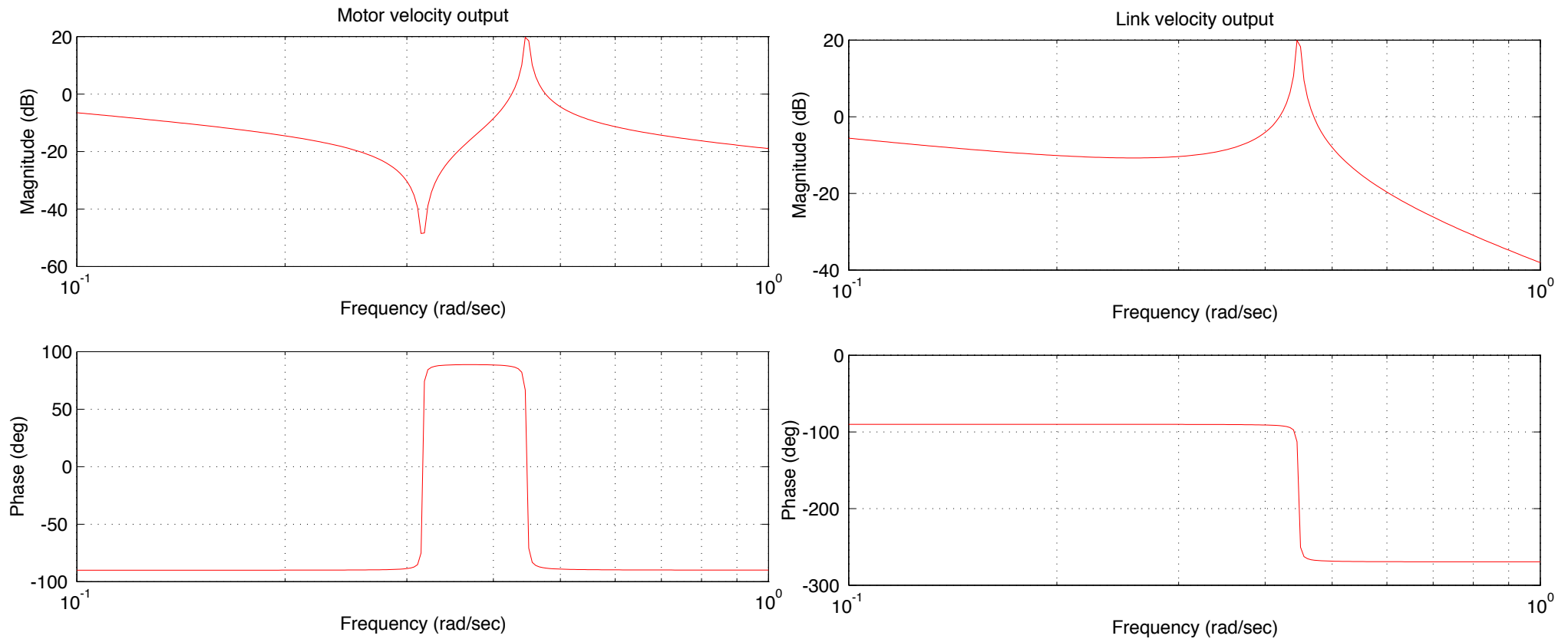
$$P_{\text{link}}(s) = \frac{q(s)}{\tau_m(s)} = \frac{K}{MBs^2 + (M + B)K} \frac{1}{s^2}$$

- NO zeros!!**
- maximum relative degree



# Single elastic joint

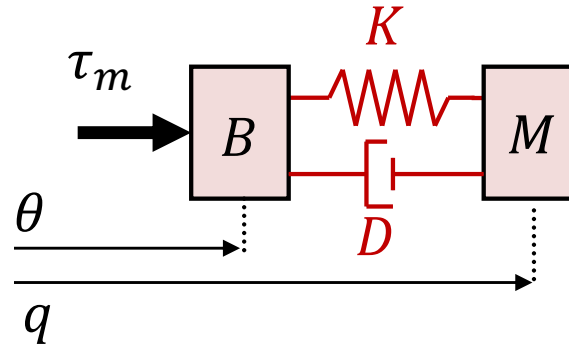
Transfer functions of interest (with some added damping...)



- typical antiresonance/resonance behavior on **motor velocity** output
- pure resonance on **link velocity** output (weak or no zeros)

# Visco-elasticity of the joints

Introduces a structural change ...



on Spong model

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) + D(\dot{q} - \dot{\theta}) \\ K(\theta - q) + D(\dot{\theta} - \dot{q}) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$

coupling type	consequence for the model
stiffness	basic static coupling, maximum relative degree (= 4) of output $q$
damping	reduced relative degree, static I/O linearization
inertia	reduced relative degree, only dynamic I/O linearization



# Regulation task

## Using a minimal PD action on motor side

for a desired **constant** link position  $q_d$

- evaluate the associated desired motor position at steady state
- collocated (**partial state**) feedback preserves passivity, with stiff  $K_\theta$  gain dominating gravity
- focus on the term for **gravity compensation** (acting on link side) from motor measurements

$$\theta_d = q_d + K^{-1}g(q_d) \qquad \tau_m = \tau_g + K_\theta(\theta_d - \theta) - D_\theta\dot{\theta}$$

$\tau_g$	gain criteria for stability	
$g(q_d)$	$\lambda_{\min} \begin{bmatrix} K & -K \\ -K & K + K_\theta \end{bmatrix} > \alpha$	[Tomei 91]
$g(\theta - K^{-1}g(q_d))$	$\lambda_{\min} \begin{bmatrix} K & -K \\ -K & K + K_\theta \end{bmatrix} > \alpha$	[De Luca, Siciliano, Zollo 04]
$g(\bar{q}(\theta)), \bar{q}(\theta): g(\bar{q}) = K(\theta - \bar{q})$	$K_\theta > 0, \lambda_{\min}(K) > \alpha$	[Ott, Albu-Schäffer 04]
$g(q) + BK^{-1}\ddot{g}(q)$	$K_\theta > 0, K > 0$	[De Luca 10]

**gravity cancellation**  
(with **full state** feedback): *more on this later...*

$$\alpha = \max\left(\left\|\frac{\partial g(q)}{\partial q}\right\|\right)$$



# Inverse dynamics

## Feedforward action for following a desired trajectory in nominal conditions

given a desired **smooth** link trajectory  $q_d(t) \in C^4$

- compute symbolically the desired **motor acceleration** and, therefore, also the desired **link jerk** (i.e., up to the fourth time derivative of the desired motion)

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$



$$\begin{aligned} \tau_{m,d} &= B\ddot{\theta}_d + K(\theta_d - q_d) \\ &= BK^{-1} \left[ M(q_d) q_d^{(4)} + 2\dot{M}(q_d) q_d^{(3)} + \ddot{M}(q_d) \ddot{q}_d + \frac{d^2}{dt^2} (C(q_d, \dot{q}_d) \dot{q}_d + g(q_d)) \right] \\ &\quad + [M(q_d) + B] \ddot{q}_d + C(q_d, \dot{q}_d) \dot{q}_d + g(q_d) \end{aligned}$$

- the inverse dynamics can be efficiently computed using a **modified Newton-Euler** algorithm (with link recursions up to the fourth order) running in  $O(N)$
- the **feedforward** command can be used in combination with a PD **feedback** control on the motor position/velocity error, so as to obtain a local but simple trajectory tracking controller



# Feedback linearization

For accurate trajectory tracking tasks

- the link position  $q$  is a **linearizing (flat) output**

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix} \iff q^{(4)} = u$$

- differentiating twice the link equation and using the motor acceleration yields

$$\tau_m = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1} \left( 2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q)) \right)$$

- an **exactly** linear and I/O decoupled closed-loop dynamics is obtained
  - to be stabilized with standard linear techniques (pole placement, LQ, ...)
- requires **higher derivatives** of  $q$   $q, \dot{q}, \ddot{q}, q^{(3)}$
- however, these can be computed **from the model** using the state measurements
- requires **higher derivatives** of the dynamics components  $\ddot{M}, \ddot{C}, \ddot{g}$
- A  $O(N^3)$  **Newton-Euler** recursive numerical algorithm is available also for this problem



# Torque control

A different set of state measurements can be used directly for tracking control

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$

$$\tau = K(\theta - q) \quad \text{measurable by a joint torque sensor}$$

$$BK^{-1}\ddot{\tau} + \tau = \tau_m - B\ddot{q} \quad \text{rewriting the motor dynamics}$$

$$\tau_m = BK^{-1}\ddot{\tau}_d + \tau_d + K_T(\tau_d - \tau) + K_S(\dot{\tau}_d - \dot{\tau}) + \alpha B\ddot{q}$$

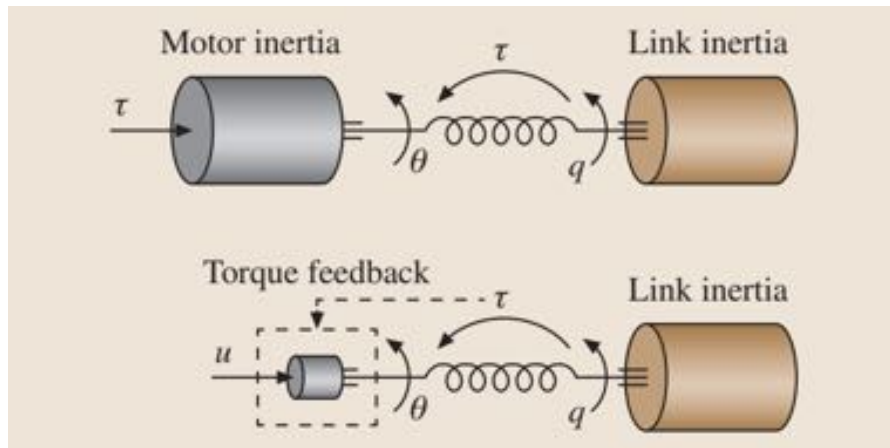
- $\alpha < 1$  for avoiding over-compensation
- useful for designing a motor side disturbance observer, e.g., to realize **friction compensation**
- basis for many **cascaded controller designs** that start from a rigid body control law  $\tau_d(q, \dot{q})$
- **higher derivatives** are still required ( $\ddot{\tau}_d, \ddot{q}$ )

# Torque feedback

An inner loop that largely reduces motor inertia and friction

consider a pure **proportional** torque feedback (+ a derivative term for the visco-elastic case)

$$\tau_m = \underbrace{BB_d^{-1}u + (I - BB_d^{-1})\tau}_{K_T} + \underbrace{(I - BB_d^{-1})DK^{-1}\dot{\tau}}_{K_S}$$



physical interpretation:

**scaling** of the motor inertia and motor friction!

[Ott, Albu-Schäffer 08]

original motor dynamics

$$B\ddot{\theta} + K(\theta - q) = \tau_m$$

visco-elastic case

$$B\ddot{\theta} + \tau + DK^{-1}\dot{\tau} = \tau_m$$



**After the torque feedback**

$$B_d\ddot{\theta} + K(\theta - q) = u$$

$$B_d\ddot{\theta} + \tau + DK^{-1}\dot{\tau} = u$$

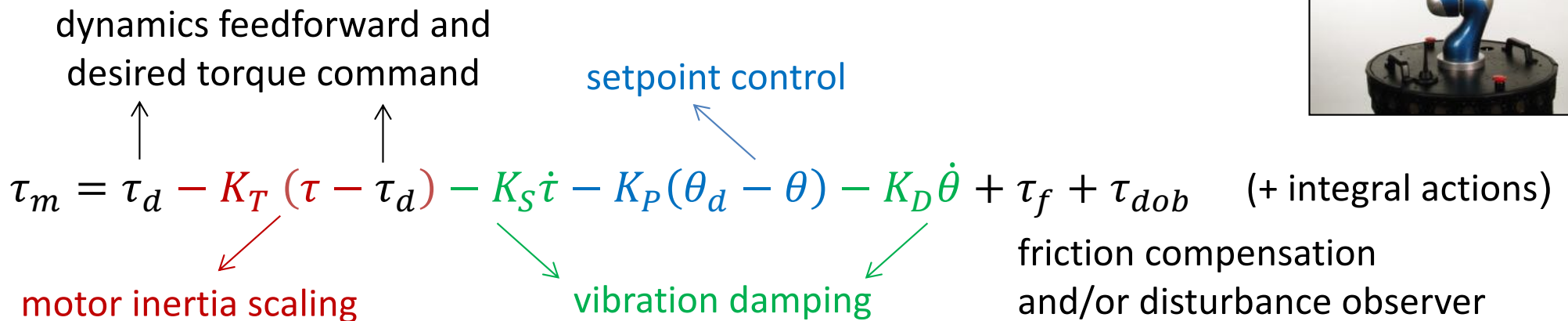


# Full-state feedback

Combining torque feedback with a motor PD regulation law

inertia scaling via torque feedback  $\tau_m = (I + K_T)u - K_T \tau - K_S \dot{\tau}$   
 regulation via motor PD, e.g. with  $u = g(\bar{q}(\theta)) + K_\theta(\theta_d - \theta) - D_\theta \dot{\theta}$

⇒ **joint level control structure** of the DLR (and KUKA) lightweight robots



## torque control

$$\begin{aligned} K_P &= 0 \\ K_D &= 0 \\ K_T &> 0 \\ K_S &> 0 \\ \tau_d & \end{aligned}$$

## position control

$$\begin{aligned} K_P &> 0 \\ K_D &> 0 \\ K_T &> 0 \\ K_S &> 0 \\ \tau_d &= g(q) \end{aligned}$$

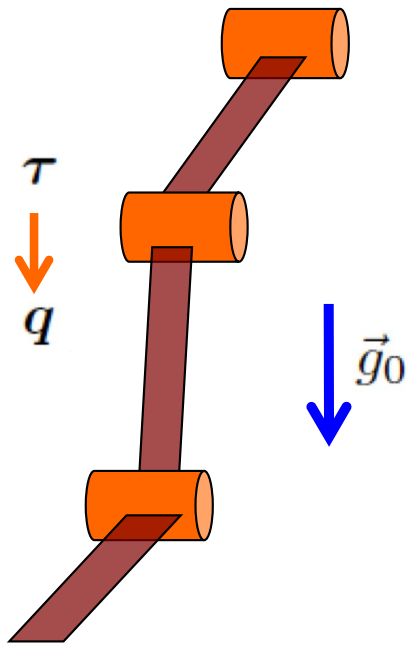
## impedance control

$$\begin{aligned} K_P &= K_T K_\theta \\ K_D &= K_T D_\theta \\ K_T &= (B B_d^{-1} - I) \\ K_S &= (B B_d^{-1} - I) D K^{-1} \\ \tau_d &= g(\bar{q}(\theta)) \end{aligned}$$

# Exact gravity cancellation

## A slightly different view

- for rigid robots this is **trivial**, due to collocation



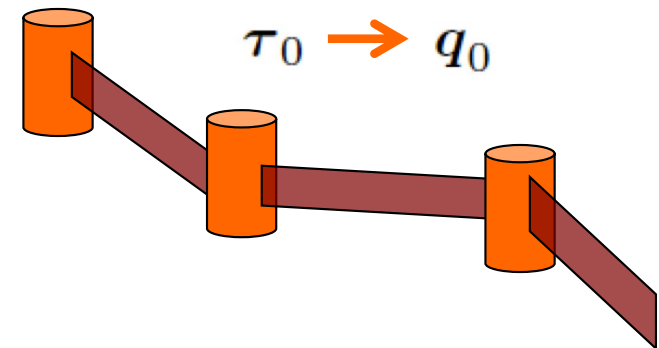
$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = \tau$$

$$\tau = \tau_g + \tau_0$$

➔

$$\tau_g = g(q)$$

$$q \equiv q_0$$

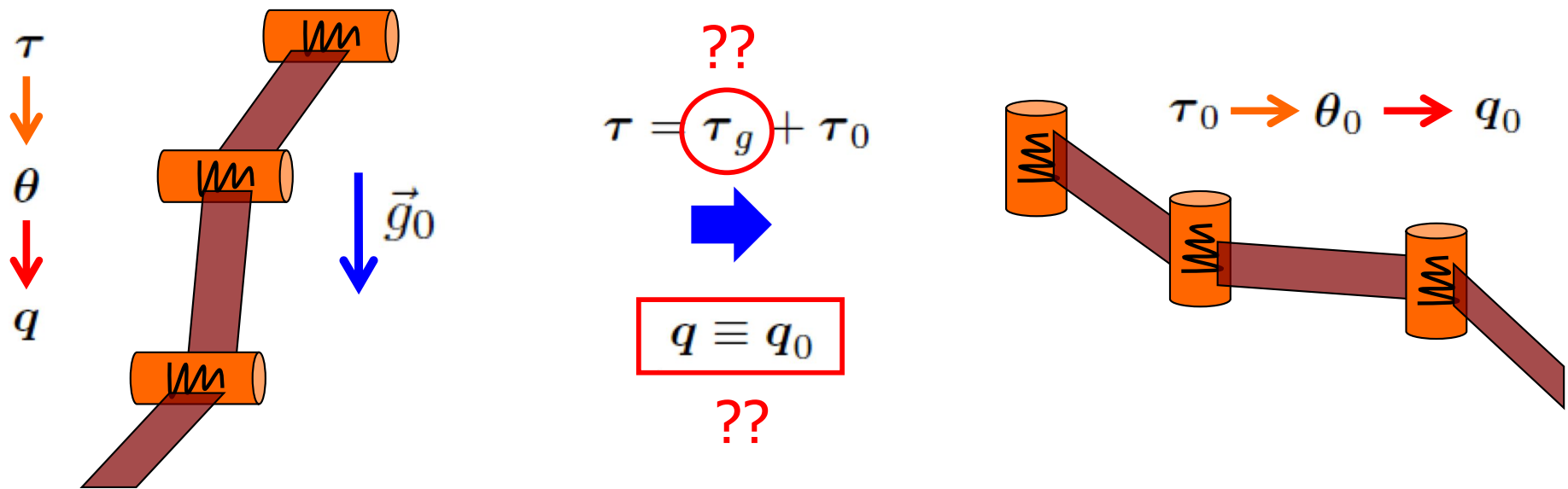


$$M(q)\ddot{q} + c(q, \dot{q}) = \tau_0$$

# Exact gravity cancellation

... based on the concept of **feedback equivalence** between nonlinear systems

- for elastic joint robots, **non-collocation** of input torque and gravity term



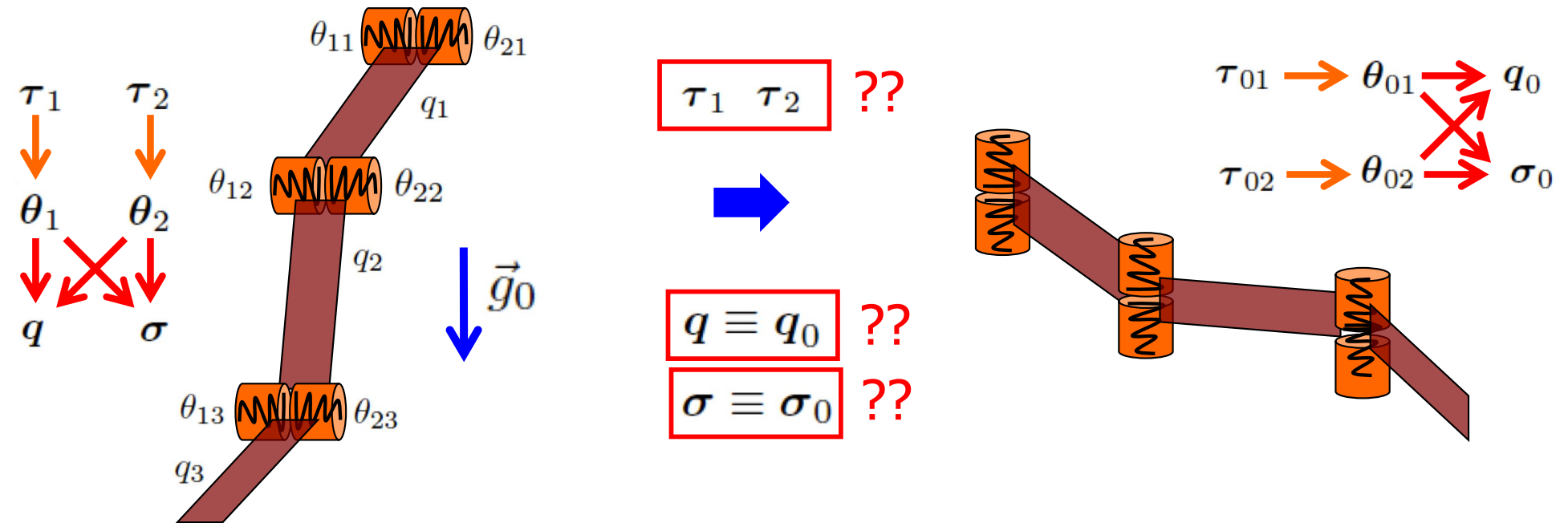
$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) + K(q - \theta) = 0$$

$$B\ddot{\theta} + K(\theta - q) = \tau$$

# Exact gravity cancellation

... generalized also to VSA robots

- same problem formulation holds also for **VSA robots** (here, in antagonistic configuration), with the additional consideration of the internal **stiffness state**



$$\phi_i = q - \theta_i$$

$$i = 1, 2$$

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) + \tau_{e1}(\phi_1) + \tau_{e2}(\phi_2) = 0$$

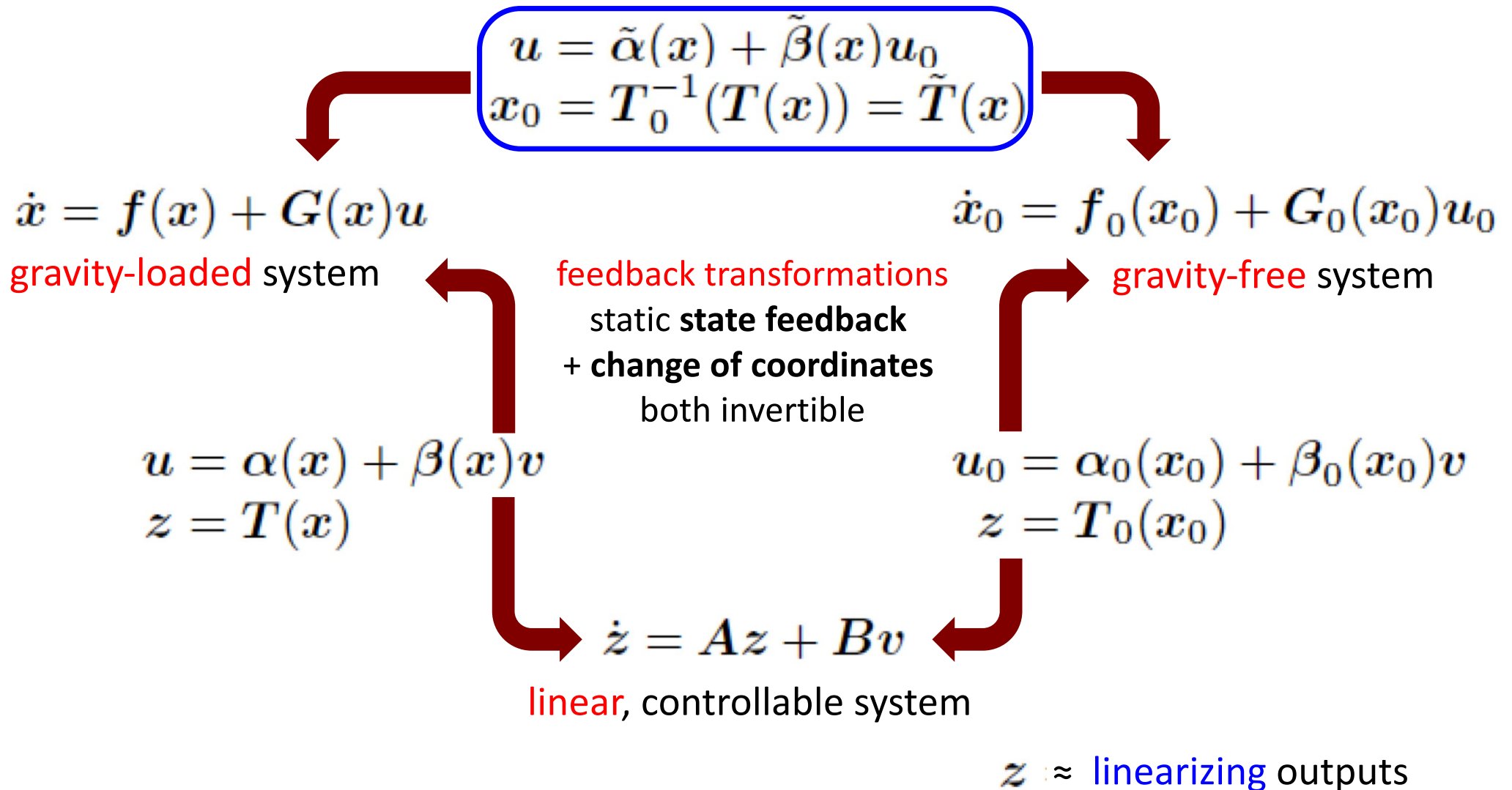
$$B_1\ddot{\theta}_1 - \tau_{e1}(\phi_1) = \tau_1$$

$$B_2\ddot{\theta}_2 - \tau_{e2}(\phi_2) = \tau_2$$



# Feedback equivalence

Exploit the system property of being feedback linearizable (without forcing it!)



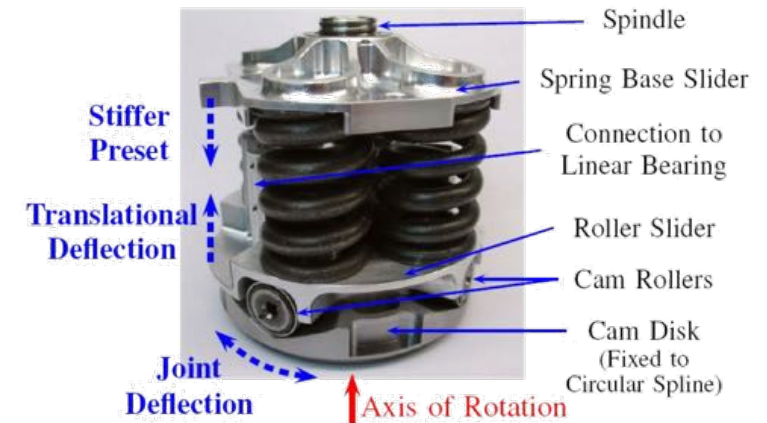
# Flexible joint robots are feedback linearizable...

... with linearizing outputs of suitable relative degrees

- robots with elastic joints
  - also with joints having **nonlinear** flexibility
- robots with VSA-based actuation
  - antagonistic VSA-II
  - serial DLR-VS joint
  - ...

linearizing output = **link position (4)**

linearizing output = **link position (4)**  
**+ joint stiffness (2)**





# Exact gravity cancellation

Elastic joint robots (including link/motor damping)

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) + D_q\dot{q} + K(q - \theta) = 0$$

$$B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) = \tau$$

$$q(t) \equiv q_0(t) \quad \forall t \geq 0 \quad \tau = \tau_g + \tau_0$$



$$\tau_g = g(q) + D_\theta K^{-1} \dot{g}(q) + B K^{-1} \ddot{g}(q)$$

$$\dot{g}(q) = \frac{\partial g(q)}{\partial q} \dot{q}$$

$$\ddot{g}(q) = \frac{\partial g(q)}{\partial q} M^{-1}(q) (K(\theta - q) - c(q, \dot{q}) - g(q) - D_q \dot{q}) + \sum_{i=1}^n \frac{\partial^2 g(q)}{\partial q \partial q_i} \dot{q} \dot{q}_i$$

requires **full state** feedback



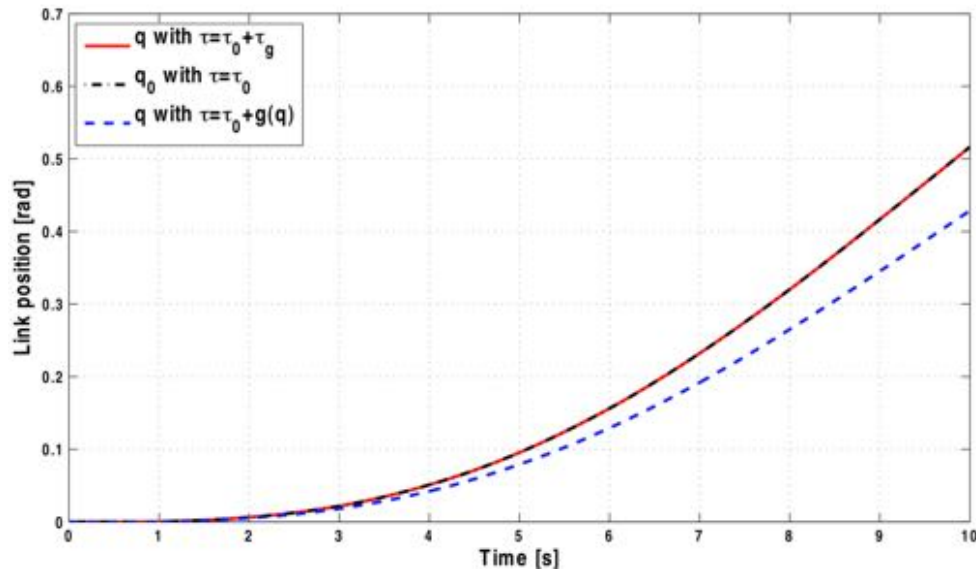
# Numerical results

Exact gravity cancellation for a **1-DOF** elastic joint

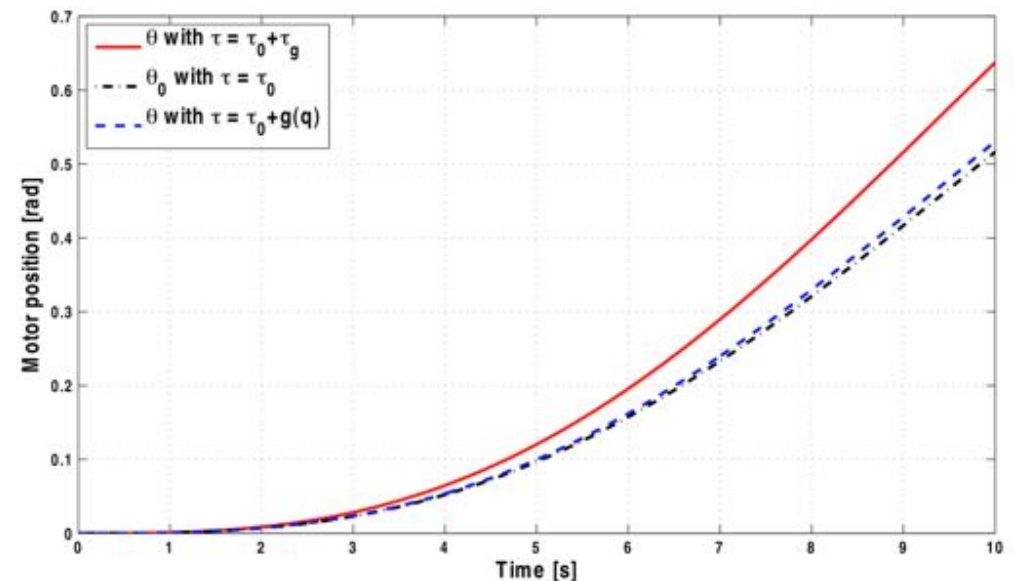
$$\tau_g = mdg_0 \left\{ \left(1 - \frac{B}{K} \dot{q}^2\right) \sin q - \frac{B}{M} \frac{mdg_0}{K} \sin q \cos q + \frac{MD_\theta - BD_q}{KM} \dot{q} \cos q + \frac{B}{M} (\theta - q) \cos q \right\}$$

$$\tau_0 = \sin 0.1\pi t$$

$$g(q) = mdg_0 \sin q$$



exact reproduction of **same link behavior**  
with and without gravity



**different motor behavior**  
with and without gravity

$$\theta = \theta_0 + K^{-1}g(q)$$





# A global PD-type regulator

Exact gravity cancellation combined with PD law on **modified** motor variables

$$\tau = \tau_g + \tau_0$$

$$\tau_g = g(q) + D_\theta K^{-1} \dot{g}(q) + BK^{-1} \ddot{g}(q)$$

$$\tau_0 = K_P(\theta_{d0} - \theta_0) - K_D \dot{\theta}_0$$

$$= K_P(q_d - \theta + K^{-1}g(q)) - K_D(\dot{\theta} - K^{-1}\dot{g}(q))$$

Global asymptotic stability can be shown using a Lyapunov analysis under “minimal” sufficient conditions (also without viscous friction)

$$K_P > 0$$

$$K > 0$$

i.e., **no** strict positive lower bounds

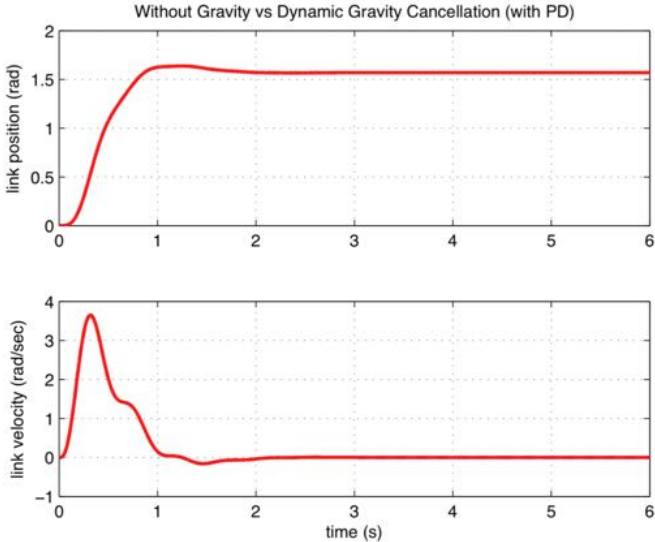
$$\text{and } K_D > 0$$



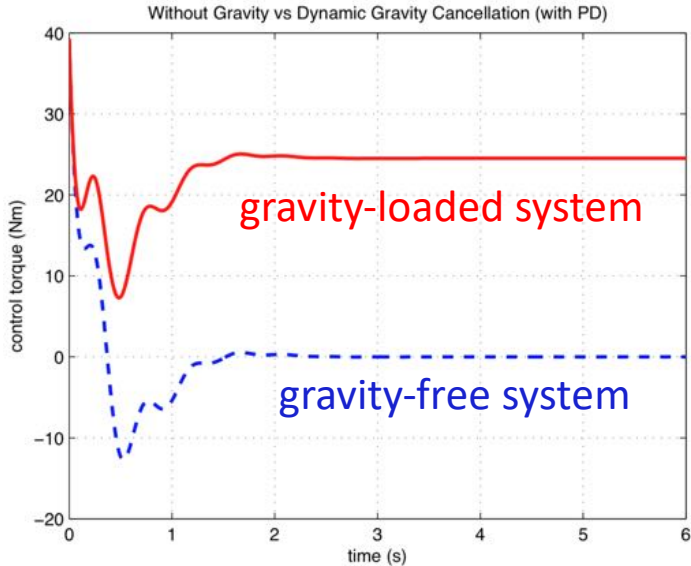
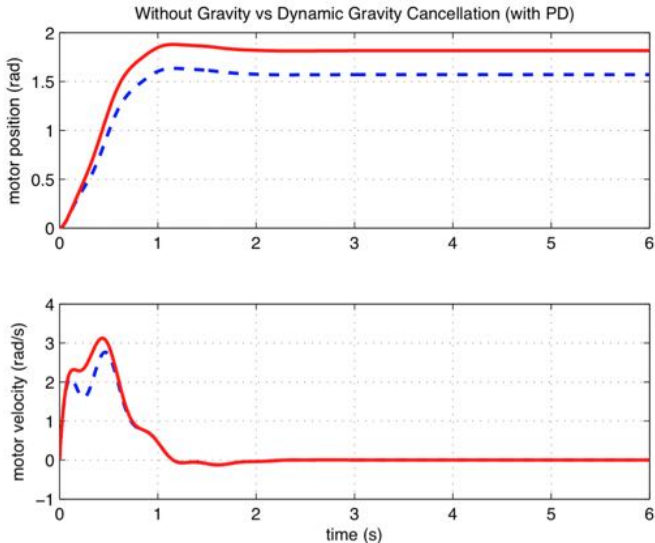
# Numerical results

## Regulation of a 1-DOF arm with elastic joint under gravity

identical link behavior



different motor behavior

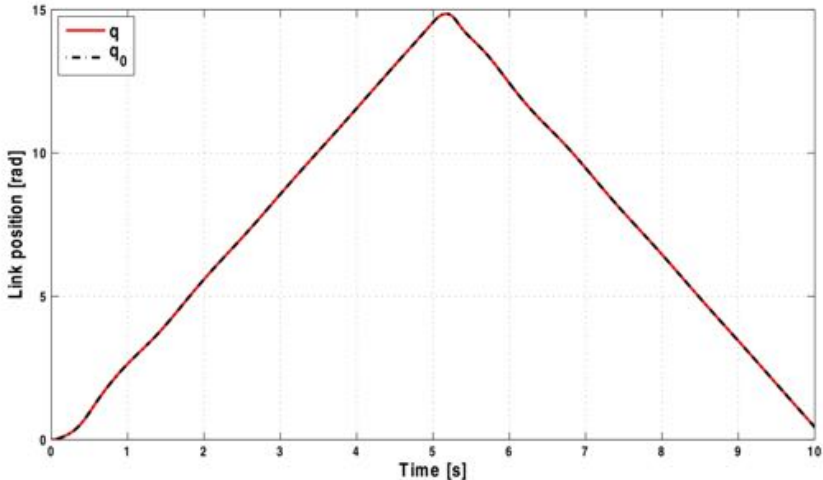
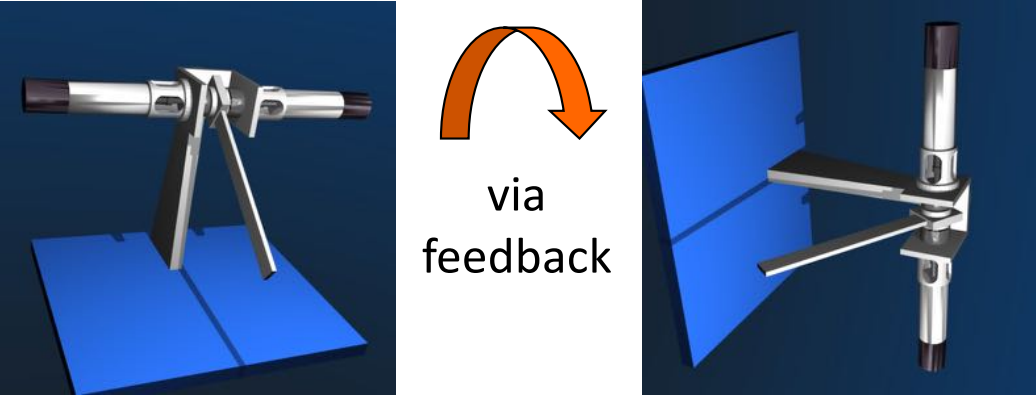


total control torque

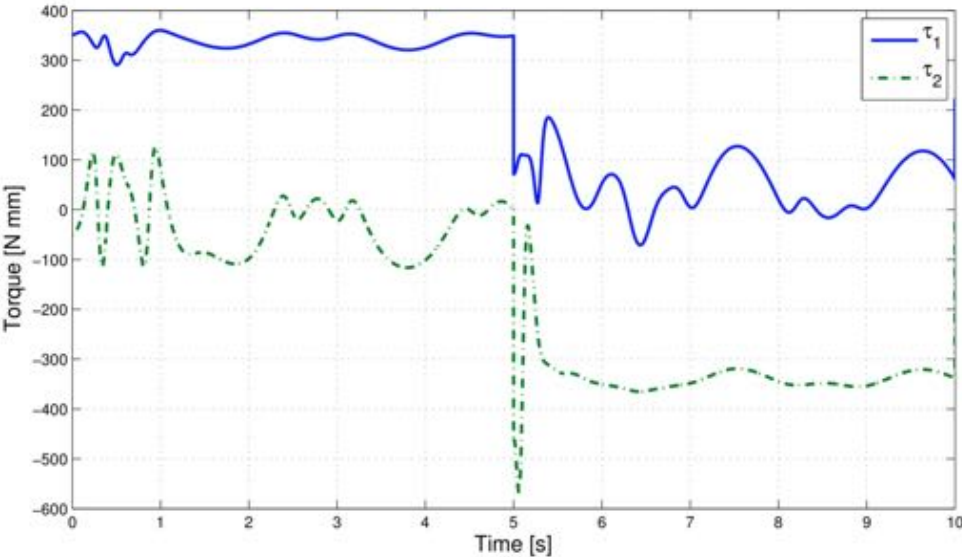
gravity-loaded system under PD + gravity cancellation  
vs.  
gravity-free system under PD (with same gains)

# Numerical results

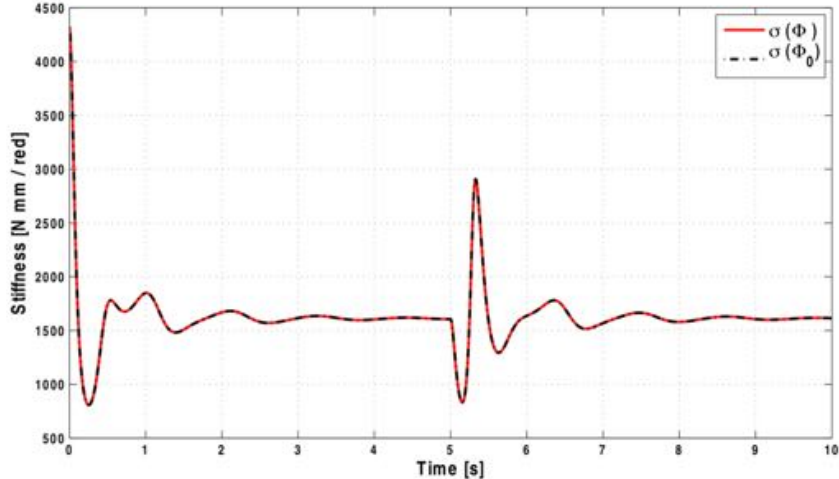
Exact gravity cancellation for the **VSA-II** of UniPisa



exact reproduction of **link behavior**



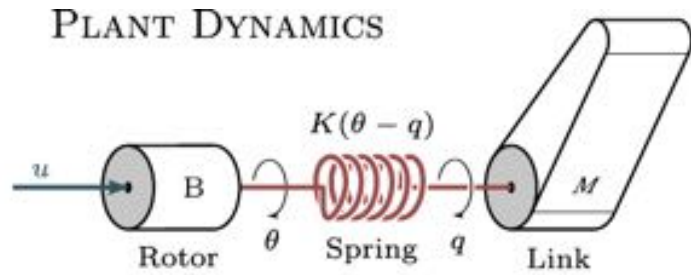
**applied torques** for gravity cancellation



exact reproduction of **stiffness behavior**

# Link vibration damping

DLR method for VSA-driven bimanual humanoid torso David [Keppler et al. 16]



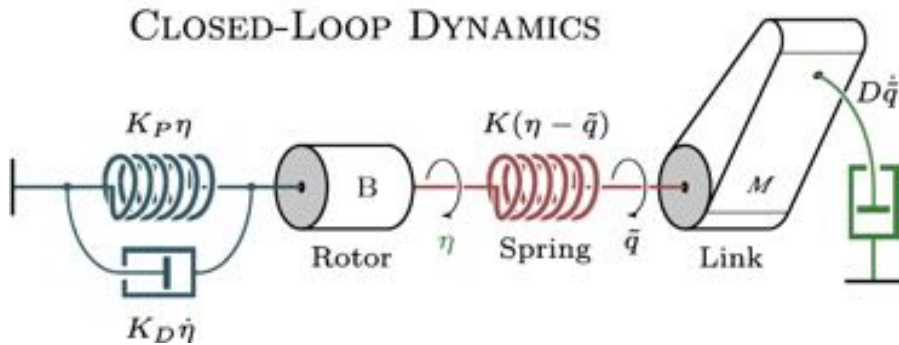
$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$

$\frac{d^2}{dt^2}$

$$K(q - \theta) = K(q - \eta) + D\dot{q}$$

state transformation

$$\tau_m = u - D\dot{q} - BK^{-1} \frac{d^2}{dt^2} D\dot{q}$$



$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\eta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \eta) \\ K(\eta - q) \end{pmatrix} = \begin{pmatrix} -D\dot{q} \\ u \end{pmatrix}$$

- same principle of **feedback equivalence** (including state transformation)
- **ESP** = Elastic Structure Preserving control
- generalizations to **trajectory tracking**, to nonlinear joint flexibility, and to viscoelastic joints



## Short outlook

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- Mature control field recently revamped by the new “explosion” of interest for compliant and soft robots
  - simpler control laws are always welcome
  - sensing requirements could be a bottleneck
  - iterative learning on repetitive tasks already in place for flexible manipulators
  
- Control ideas assessed for concentrated elasticity at the joints can migrate to other classes of soft-bodied manipulators
  - but intrinsic constraints and control limitations should be kept in mind (e.g., instabilities in the system inversion of tip trajectories for flexible link robots)
  
- Emerging notion: not fighting against the natural dynamics!
  - and trying also not to give up too much of the desirable performance ...